An explanation of under-diversification puzzles through ambiguity tastes and beliefs

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Abstract

We develop a portfolio selection model with heterogenous correlated returns ambiguity and find that ambiguity tastes inferred from investor decisions and beliefs estimated from market data explain the equity home bias puzzle globally. Under worst-case ambiguity aversion, we explain the home bias by beliefs consistent with the market estimates. We also explain the bias by ambiguity aversion averaging about 0.6 globally, closely matching current experimental estimates from US, Dutch, and Chinese population samples. A model prediction under homogeneous ambiguity is verified on a large dataset of US household portfolios, and we document ambiguity as a determinant of household under-diversification.

Keywords: Equity home bias puzzle, household under-diversification, ambiguity aversion, ellipsoidal ambiguity sets.

JEL Classification: C61, D81, G11, G15, G41, G51.

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1 Introduction

The *equity home bias puzzle* refers to the observation that investors tend to over-invest in domestic equities (Cooper and Kaplanis, 1994; French and Poterba, 1991; Tesar and Werner, 1995) despite the benefits of international diversification under the ICAPM (Adler and Dumas, 1983). Ambiguity aversion has been shown to induce home bias, but a complete puzzle explanation is missing. We develop a portfolio selection model with *perceived ambiguity* as a characteristic of investors' beliefs and *ambiguity aversion* as a behavioral characteristic of their tastes (Baillon, Huang, Selim, and Wakker, 2018). Taken to the data, the model explains the equity home bias globally through both channels. A related puzzle is *household under-diversification* (Friend and Blume, 1975). A model prediction that perceived ambiguity induces household under-diversification is verified on a large dataset of US household portfolios (Barber and Odean, 2000), and a regression with controls documents ambiguity as a determinant of this puzzle as well.

We model heterogenous ambiguities for the home and foreign markets using *ellipsoidal sets* (Ben-Tal, El Ghaoui, and Nemirovski, 2009). The ellipsoids with correlated returns ambiguity are salient. Depending on the correlations, the home allocation can be zero even if home returns are less ambiguous than the foreign or positive even if more ambiguous. The model is unbiased, with a non-mechanical relation between ambiguity and diversification. Ceteris paribus, bias increases with ambiguity like in existing models, but correlations in the ambiguity set nest alternative possibilities to let the data speak.

We introduce correlated returns heterogeneous ambiguity starting from the continuous ambiguity aversion α-maxmin model of Ghirardato, Maccheroni, and Marinacci (2004). We first obtain a worst-case model optimizing a non-parametric performance ratio without a normality assumption. This model is parsimonious, without free parameters of risk or ambiguity aversion. We show that it satisfies second-order stochastic dominance consistency, abstracting from investor risk aversion parameters. It also satisfies the maxmin axioms of Gilboa and Schmeidler (1989), abstracting from ambiguity aversion parameters. We generalize the model to vary from worst-case ambiguity aversion to best-case ambiguity-seeking, with ambiguity neutrality in between. The worst-case model can explain the puzzle through ambiguity beliefs, and the general model through tastes.

We put the models to the data of 21 developed and 19 emerging markets, estimating ellipsoidal ambiguity sets from market data using different methods. The worst-case model optimal allocations match the observed home bias for perceived ambiguity within the market estimates for all countries. Interval ambiguity sets without correlations do not match the observed bias. Using the general model, we infer the ambiguity aversion that matches the observed bias under the market-estimated perceived ambiguity. We obtain ambiguity aversion averaging 0.6 with little cross-country variability and statistically larger than the neutral 0.5. Our estimate is within the range of experimental estimates (0.52-0.63) from Dutch (Dimmock, Kouwenberg, and Wakker, 2016b), US (Dimmock, Kouwenberg, Mitchell, and Peijnenburg, 2016a), or Chinese (Potamites and Zhang, 2012) population samples.

Cooper, Sercu, and Vanpée (2012) point out that the challenge in explaining the puzzle is to generate allocations matching the observed bias with realistic parameter values. Our models pass this test for forty countries with different ambiguity estimation methods.

Extensive literature has sought to explain the equity home bias puzzle (Cooper et al., 2012). The survey by Coeurdacier and Rey (2013) identifies potential explanations through (i) real exchange rate and non-tradable income risk, (ii) trading costs in international markets, such as transaction costs, differences in tax treatments between national and foreign assets or differences in legal frameworks, and (iii) informational frictions and behavioral biases. Tesar and Werner (1995) anticipated that an explanation would require a rich portfolio analysis model, with Glassman and Riddick (2001) developing such a model to identify adjustments that could explain the puzzle and finding that no single set of adjustments suffices. One important strand of research looks at ambiguity (Knight, 1921) as a potential explanation, establishing a relation in theoretical models and finding evidence in laboratory setups and survey data. However, a full explanation is still missing; this is the direction we follow and fill the gap.

Theoretical works by Dow, Da, and Werlang (1992) and Epstein and Wang (1994) show, respectively, that ambiguity aversion leads to price interval with zero holdings in an ambiguous asset and establish the existence of an equilibrium process with possibly indeterminate equilibria. The authors suggest that their findings may explain the puzzles.

Recent models build on these early works to show that ambiguity induces home bias or under-diversification. However, either they did not put the models to the data, or when they did the test, they reached only a partial explanation. Epstein and Miao (2003) develop a two-asset model with heterogeneous ambiguities and show that ambiguity tilts the portfolios towards home but does not resolve the puzzle in US data. Uppal and Wang (2003) show that ambiguity induces under-diversification, but tests on three markets do not match the observed allocations for reasonable ambiguity parameters.[1] Baele, Pungulescu, and Ter Horst (2007) introduce ambiguity in mean-variance asset allocation and obtain portfolios halving, but not eliminating, the bias. Cao, Han, Hirshleifer, and Zhang (2011) develop a two-asset model to explain observed allocations for reasonable "fear of unfamiliarity," and Boyle, Garlappi, Uppal, and Wang (2012) develop a model with interval ambiguity avoidance that generates flight to familiarity in controlled experiments.

[1] We implemented their model with recent data (1999-2019) and also found that the generated portfolios do not match the observed ones.

The work closest to ours is Maccheroni, Marinacci, and Ruffino (2013), who developed a mean-variance model with smooth ambiguity aversion and showed that ambiguity aversion reduces the optimal exposure to ambiguity with the portfolio composition depending on the ambiguous asset's alpha with the risky asset as the benchmark. We add tastes, beliefs, and correlations in the ambiguity sets, do away with the normality assumption, and conduct extensive tests explaining the puzzle on a large sample of countries. In the process, we also obtain estimates of ambiguity aversion for a large sample of countries that closely match existing experimental evidence from US, Dutch, and Chinese population samples.

To put the models to the data, we need the asset perceived ambiguities (Bianchi and Tallon, 2019) and investor ambiguity aversion (Baillon et al., 2018). Dlugosch and Wang (2022) put a model with perceived ambiguity to the data and show on a sample of twenty-three countries that home ambiguity aversion correlates positively with foreign bias, accounting for several controls, but does not match the observed allocations. Hara and Honda (2022) allow short sales in a mean-variance smooth ambiguity aversion model and obtain the optimality conditions involving the means, variances, and covariances of asset returns, investor attitudes toward risk and ambiguity, and perceived ambiguity, and show that ambiguity aversion explains the market capitalization and why the value-weighted portfolio is not the tangency portfolio. This work suggests that a potential explanation of the puzzle must consider the assets' mean returns and higher moments, including correlations, perceived ambiguity, and investor ambiguity and risk aversion. This is what our models achieve.

Ambiguity-averse investors were shown to under-diversify in experimental settings by Ahn, Choi, Gale, and Kariv (2014); Bossaerts, Ghirardato, Guarnaschelli, and Zame (2010). A relation between ambiguity aversion and home bias and portfolio under-diversification was documented in survey data by Bianchi and Tallon (2019); Dimmock et al. (2016a), and Dimmock et al. (2016b) found that the relation holds when investors perceive high asset ambiguity. Peijnenburg (2018) found that reducing ambiguity through learning ameliorates under-diversification. Both beliefs and tastes matter in experiments.

We contribute to the literature a novel model with ambiguity tastes and beliefs and a comprehensive empirical study explaining the equity home bias through both channels.

Methodologically, our model captures return correlations in the ambiguity sets, whereas earlier works with interval sets do not.[2] Following the literature on robust optimization (Mulvey, Vanderbei, and Zenios, 1995) we use ellipsoidal ambiguity sets (Ben-Tal and Nemirovski, 1998) to incorporate the returns' covariance matrix. Like earlier works, we

[2]See, e.g., Bianchi and Tallon (2019); Boyle, Garlappi, Uppal, and Wang (2012); Chen and Epstein (2002); Dlugosch and Wang (2022); Epstein and Miao (2003); Epstein and Wang (1994); Klibanoff, Marinacci, and Mukerji (2005); Maccheroni, Marinacci, and Ruffino (2013); Uppal and Wang (2003).

show that under-diversification increases with perceived ambiguity, but we nest alternative possibilities by interacting ambiguity with correlations. The model optimizes a performance ratio for stable distributions (*mean-to-Conditional Value-at-Risk*, Farinelli, Ferreira, Rossello, Thoeny, and Tibiletti (2008); Rachev, Stoyanov, and Fabozzi (2008)), and we prove second-order stochastic dominance (SSD) consistency for the worst-case. Our models are new to the literature.[3]

Empirically, our study on a large sample of developed and emerging markets with ambiguity sets estimated from market data provides a complete explanation of the equity home bias through beliefs and tastes. Perceived ambiguity, much below the market estimates, leads to the observed bias under worst-case ambiguity aversion. Mild ambiguity aversion under the market-estimated ambiguity also leads to the observed bias. These explanations are complementary. A significant negative result is to show that models without correlation in the ambiguity set do not explain the puzzle.

We also inform the literature on the determinants of household under-diversification. Our worst-case model for a homogeneous ambiguity set predicts that less diversified portfolios are less ambiguous. We verify this prediction on US household portfolios from the database of Barber and Odean (2000) and document ambiguity as an economically and statistically significant determinant. Earlier works identify a preference for skewness (Mitton and Vorkink, 2007), local bias (Goetzmann and Kumar, 2008), behavioral (Barber and Odean, 2008), and ambiguity aversion (Dimmock et al., 2016b), and we add perceived ambiguity to this list.

2 Portfolio selection with ambiguity tastes and beliefs

We select portfolios with the highest attainable performance ratio $\mathcal{R}(\tilde{r}_p)$, where $\tilde{r}_p$ is the random variable of portfolio returns. $\mathcal{R}$ combines reward and risk in a single ratio, and portfolios are selected based on security characteristics without specifying investor preferences (Farinelli et al., 2008). For instance, Pedersen, Fitzgibbons, and Pomorski (2021) maximize the Sharpe ratio for investors with a quadratic utility function to introduce non-pecuniary (ESG) criteria. We choose an SSD consistent performance ratio (see definitions in Appendix C.1) to account for the skewed returns of the international markets (Ghysels, Plazzi, and Valkanov, 2016). Maximizing an SSD criterion for investors with concave, non-decreasing utility functions, we can introduce the additional criteria of ambiguity tastes and beliefs.

The portfolio return is given by $\tilde{r}_p = \tilde{r}^\top w$, where $w \in \mathbb{X} \subset \mathbb{R}^n$ is the vector of asset

[3]See the recently published Bertsimas and Den Hertog (2022); Rahimian and Mehrotra (2022).

weights and $\tilde{r} \in \mathbb{R}^n$ is the vector of random asset returns with expected value $\bar{r}$. Excess returns are over the risk-free rate r_f. $\mathbb{X} = \{w \in \mathbb{R}^n \mid w \geq 0,\ e^\top w = 1\}$ is the set of feasible portfolios with no short sales, with $e^\top$ a conformable vector of ones. To differentiate home (h) and foreign (f) assets, we take $w = (w_h, w_f)$ as the concatenation of home and foreign allocations $w_h \in \mathbb{R}$ and $w_f \in \mathbb{R}^{n-1}$, with conformable random returns $\tilde{r}_h$ and $\tilde{r}_f$.

We introduce ambiguity tastes and beliefs, with mean returns $\bar{r}$ from an ambiguity set U with probability distribution $\pi \in \mathbb{D}$ (beliefs) and investor ambiguity aversion denoted by parameter λ (tastes). The model of Ghirardato et al. (2004) is written as

$$\max_{w \in \mathbb{X}} \ \lambda \left(\min_{\bar{r} \in U} \min_{\pi \in \mathbb{D}} \mathcal{R}(\tilde{r}_p) \right) + (1 - \lambda) \left(\max_{\bar{r} \in U} \max_{\pi \in \mathbb{D}} \mathcal{R}(\tilde{r}_p) \right). \tag{1}$$

$\lambda = 1$ corresponds to the *worst-case* ambiguity aversion (Gilboa and Schmeidler, 1989), $\lambda = 0$ corresponds to best-case ambiguity seeking, and $\lambda = 0.5$ to ambiguity neutrality.

For a performance ratio without the normality assumption, we use the conditional value-at-risk (CVaR) tail risk measure as the negative of the expected value of excess returns below a threshold; see Appendix Definition C.1. This measures losses.[4] We maximize the *mean-to-CVaR* (MtC) ratio of portfolio expected excess returns divided by the CVaR of losses (Farinelli et al., 2008; Rachev et al., 2008). The unambiguous portfolio selection model is given by

$$\max_{w \in \mathbb{X}} \mathcal{R} \doteq \text{MtC} = \frac{\mathbb{E}(\tilde{r}_p - r_f)}{\text{CVaR}(\tilde{r}_p - r_f)}. \tag{2}$$

We define ambiguity and introduce (2) into the general model (1). We assume that the joint probability distribution of returns belongs to the class of distributions with means $\bar{r} \in \mathbb{R}^n$ and covariance matrix Σ from the space of positive definite matrices.

Definition 2.1 (Ambiguity in distribution)**.** *The distribution of $\tilde{r}$ is of the form*

$$\mathbb{D} = \{\pi \mid \mathbb{E}[\tilde{r}] = \bar{r},\ Cov[\tilde{r}] = \Sigma\}.$$

We consider ambiguous means and known covariance since portfolio sensitivity to mean estimation errors is an order of magnitude larger than to covariance errors (Britten-Jones, 1999; Broadie, 1993; Kaut et al., 2007) and because we can improve the accuracy of second moments estimates by increasing the observation frequency (Cao et al., 2011).[5]

[4]CVaR coincides with the coherent risk measure of Artzner, Delbaen, Eber, and Heath (1999) for continuous distributions. The definition for general distributions, including the discrete distributions in this paper, is due to Rockafellar and Uryasev (2002). Portfolio models with CVAR include Alexander, Coleman, and Li (2006); Huang, Zhu, Fabozzi, and Fukushima (2008); Kaut, Wallace, Vladimirou, and Zenios (2007); Mausser and Romanko (2018).

[5]Portfolio models with ambiguous covariance include El Ghaoui et al. (2003); Goldfarb and Iyengar

The means are known only to the extent that they belong to an ellipsoidal ambiguity set.

Definition 2.2 (Ellipsoidal correlated returns ambiguity)**.** *Mean returns belong to the ellipsoidal set*

$$U = \{\bar{r} \in \mathbb{R}^n \mid (\bar{r} - \hat{r})^\top \hat{\Sigma}^{-1} (\bar{r} - \hat{r}) \leq \delta^2\},$$

where $\hat{r}$ is the center of the ellipsoid with size δ, and $\hat{\Sigma}$ is the covariance matrix estimate.

Ellipsoidal ambiguity sets account for the correlation between assets, unlike the interval ambiguity sets. Their size measures perceived ambiguity, with larger ellipsoids signifying higher ambiguity.

We introduce *heterogenous* ambiguity sets for home and foreign:

$$U_h = \{\bar{r}_h \in \mathbb{R} \mid \left(\frac{\bar{r}_h - \hat{r}_h}{\hat{\sigma}_h}\right)^2 \leq \delta_h^2\}, \tag{3}$$

$$U_f = \{\bar{r}_f \in \mathbb{R}^{n-1} \mid (\bar{r}_f - \hat{r}_f)^\top \hat{\Sigma}_f^{-1} (\bar{r}_f - \hat{r}_f) \leq \delta_f^2\}, \tag{4}$$

where $\hat{\sigma}_h$ is the home standard deviation and $\hat{\Sigma}_f$ is the covariance matrix of foreign.[6]

Substituting (2) into (1), we write the complete portfolio selection model:

$$\max_{(w_h, w_f) \in \mathbb{X}} \lambda \left(\min_{\bar{r}_h \in U_h, \bar{r}_f \in U_f} \min_{\pi \in \mathbb{D}} \frac{\mathbb{E}(\tilde{r}_p - r_f)}{\text{CVaR}(\tilde{r}_p - r_f)} \right) + (1 - \lambda) \left(\max_{\bar{r}_h \in U_h, \bar{r}_f \in U_f} \max_{\pi \in \mathbb{D}} \frac{\mathbb{E}(\tilde{r}_p - r_f)}{\text{CVaR}(\tilde{r}_p - r_f)} \right). \tag{5}$$

Using the worst-case model ($\lambda = 1$), we test whether ambiguity beliefs explain the puzzle. We start with the home perceived ambiguity estimated from market data, compute the foreign ambiguity set size that produces optimal solutions matching the observed home bias, and check whether it aligns with the market estimates of foreign perceived ambiguity. With continuous ambiguity aversion, we take the market-estimated home and foreign ambiguities as given and check whether a (reasonable) parameter λ exists with an optimal solution matching the observed home bias. This tests whether ambiguity tastes explain the puzzle. For robustness, we use two fundamentally different methods to obtain the market-estimated ambiguities.

To solve (5), we need analytical solutions to the worst- and best-case inner optimization problems. We formulate the worst-case inner problem in subsection 2.1, building on Chen, He, and Zhang (2011); Lotfi and Zenios (2018). The best-case inner problem can be unbounded unless we discipline the model with a suitable returns distribution and

(2003). We can extend our model for a homogeneous ambiguity set to joint ambiguity of means and covariance, albeit such an extension is not evident for heterogeneous sets.

[6]The home ambiguity set for a single asset reduces to an interval but can be easily extended to ellipsoids for multiple home assets, e.g., for the eurozone. The two ambiguity sets are assumed to be independent.

develop the general model for a multivariate t-student distribution in subsection 2.2.

2.1 Model with perceived ambiguity

For $\lambda = 1$, (5) is a model with perceived ambiguity only. We show it to be SSD consistent, obtain a computationally tractable formulation, and derive analytical conditions on correlations and ambiguity that induce bias and under-diversification for the case of two assets. Importantly, we demonstrate the significance of ellipsoidal vs interval ambiguity sets. For a homogeneous ambiguity set, the model predicts that lower ambiguity implies less diversified portfolios.

2.1.1 Worst-case portfolio selection

For $\lambda = 1$ we have the worst-case MtC maximization model

$$\max_{(w_h, w_f)\in\mathbb{X}} \min_{\bar{r}_h\in U_h, \bar{r}_f\in U_f} \min_{\pi\in\mathbb{D}} \frac{\mathbb{E}(\tilde{r}_p - r_f)}{\mathrm{CVaR}(\tilde{r}_p - r_f)}. \tag{6}$$

We establish SSD consistency under the reasonable assumption that a portfolio exists with positive worst-case excess returns and losses.

Theorem 2.1 (Second order stochastic dominance of worst-case MtC portfolios)**.** *Let $\mathbb{X}_+$ denote the space of all feasible portfolios that have positive worst-case mean excess return and worst-case CVaR over the ambiguity sets $\mathbb{D}$, U_h, and U_f. Then, the worst-case MtC is SSD consistent for all portfolios in $\mathbb{X}_+$.* (See Appendix C.2 for the proof.)

The worst-case model is formulated as a second-order cone program solvable with interior point methods (Ben-Tal et al., 2009). This is our work-horse.

Theorem 2.2 (Second-order cone program worst-case model)**.** *Model (6) is cast as:*

$$\begin{aligned}
\max_{w'_h\in\mathbb{R}_+, w'_f\in\mathbb{R}^{n-1}_+} \quad & ((\hat{r}_h - r_f) - \delta_h\hat{\sigma}_h)\, w'_h + \left((\hat{r}_f - r_f e)^\top w'_f - \delta_f\sqrt{{w'}_f^\top\hat{\Sigma}_f w'_f}\right) \qquad (7)\\
\text{s.t.} \quad & -((\hat{r}_h - r_f) - \delta_h\hat{\sigma}_h)\, w'_h - \left((\hat{r}_f - r_f e)^\top w'_f - \delta_f\sqrt{{w'}_f^\top\hat{\Sigma}_f w'_f}\right)\\
& + \frac{\sqrt{\alpha}}{\sqrt{1-\alpha}}\sqrt{{w'}_h^2\hat{\sigma}_h^2 + 2w'_h\hat{\sigma}_{hf}^\top w'_f + {w'}_f^\top\hat{\Sigma}_f w'_f} \leq 1\\
& w'_h + e^\top w'_f > 0.
\end{aligned}$$

From the optimal solution ${w'}^\star_h$ and ${w'}^\star_f$, we obtain the solution to (6) as $w^\star_h = \frac{1}{e\top({w'}^\star_h + {w'}^\star_f)}{w'}^\star_h$ and $w^\star_f = \frac{1}{e\top({w'}^\star_h + {w'}^\star_f)}{w'}^\star_f$. (See Appendix D.1 for the proof.)

$(\hat{r} - r_f)$ is the risk premium and $(\hat{r} - r_f) - \delta\hat{\sigma}$ is the *ambiguity-adjusted (aa-) premium.*

2.1.2 The case of two assets

We obtain the two-asset solution for correlation ρ and ambiguity sets

$$\begin{aligned} U_h &= \{\bar{r} \in \mathbb{R} \mid \frac{|\bar{r} - \hat{r}_h|}{\hat{\sigma}_h} \leq \delta_h\}, \\ U_f &= \{\bar{r} \in \mathbb{R} \mid \frac{|\bar{r} - \hat{r}_f|}{\hat{\sigma}_f} \leq \delta_f\}, \end{aligned} \tag{8}$$

to study the ambiguity effect on home allocation and diversification. From Theorem 2.2, we obtain the second-order cone program (Appendix C.3) and give its solutions in the following theorem, under the reasonable assumption that at least one of the aa-premia is positive. (The case of both aa-premia negative is uninteresting as it assumes no zero-premium (risk-free) asset.)

Theorem 2.3 (Optimal allocation with positive correlation)**.** *For $\rho > 0$, and assuming at least one positive aa-premium, the optimal solution for (6) with assets $w_h, w_f \in \mathbb{R}$ is:*

i. For $\rho(s_f - \delta_f) < (s_h - \delta_h) < \frac{1}{\rho}(s_f - \delta_f)$,

$$w_h^\star = \frac{((s_h - \delta_h) - \rho(s_f - \delta_f))\,\hat{\sigma}_f}{((s_f - \delta_f) - \rho(s_h - \delta_h))\,\hat{\sigma}_h + ((s_h - \delta_h) - \rho(s_f - \delta_f))\,\hat{\sigma}_f}, w_f^\star = 1 - w_h^\star. \tag{9}$$

ii. $w_h^\star = 0$ for $(s_h - \delta_h) \leq \rho(s_f - \delta_f)$, and $w_h^\star = 1$ for $(s_h - \delta_h) \geq \frac{1}{\rho}(s_f - \delta_f)$.

(See Appendix C.3 for the proof.)

Here, s denotes the Sharpe ratio, and we refer to the Sharpe ratio with ambiguity-adjusted risk premium $(s - \delta)$ as the *ambiguity-adjusted (aa-) Sharpe ratio*. It follows from this theorem that for positive correlation, allocating everything to the home asset is optimal if its correlation-scaled aa-Sharpe is higher than the aa-Sharpe of the foreign.

For negative correlation, we always have $\rho(s_f - \delta_f) < (s_h - \delta_h) < \frac{1}{\rho}(s_f - \delta_f)$. Following the proof of the theorem, we arrive at case i, and corner solutions are ruled out.

The following corollary gives the ambiguity effect on the optimal allocation.

Corollary 2.1. *The partial derivatives of $w_h^\star$ with respect to δ_h and δ_f are:*

i. If $\rho(s_f - \delta_f) < (s_h - \delta_h) < \frac{1}{\rho}(s_f - \delta_f)$

$$\frac{\partial w_h^\star}{\partial \delta_h} = \frac{\hat{\sigma}_h \hat{\sigma}_f (1 - \rho^2)(\delta_f - s_f)}{[((s_f - \delta_f) - \rho(s_h - \delta_h))\,\hat{\sigma}_h + ((s_h - \delta_h) - \rho(s_f - \delta_f))\,\hat{\sigma}_f]^2}, \tag{10}$$

$$\frac{\partial w_h^\star}{\partial \delta_f} = \frac{\hat{\sigma}_h \hat{\sigma}_f (1 - \rho^2)(s_h - \delta_h)}{[((s_f - \delta_f) - \rho(s_h - \delta_h))\,\hat{\sigma}_h + ((s_h - \delta_h) - \rho(s_f - \delta_f))\,\hat{\sigma}_f]^2}. \tag{11}$$

ii. Zero, if $(s_h - \delta_h) \leq \rho(s_f - \delta_f)$ *or* $(s_h - \delta_h) \geq \frac{1}{\rho}(s_f - \delta_f)$.

(See Appendix C.4 for the proof.)

The theorem and its corollary show how the worst-case model can explain the home bias in a two-country setting. The optimal allocation between an unambiguous home and an ambiguous foreign asset is obtained from Theorem 2.3 for $\delta_h = 0, \delta_f > 0$. It depends on the foreign asset's perceived ambiguity and the means, standard deviations, and correlation of home and foreign. It may be optimal to allocate everything to the home if its correlation-adjusted aa-Sharpe is higher than the aa-Sharpe of the foreign (Theorem case *ii*). For middle solutions, the derivative of $w_h^\star$ with respect to the foreign ambiguity is positive for $\rho(s_f - \delta_f) < (s_h - \delta_h) < \frac{1}{\rho}(s_f - \delta_f)$ and zero otherwise. Hence, increasing foreign ambiguity could tilt the portfolio towards home depending on the rewards, risk, correlations, and perceived ambiguity. It may be optimal for the home allocation to deviate from the ICAPM benchmark. Similar arguments apply for $\rho < 0$.

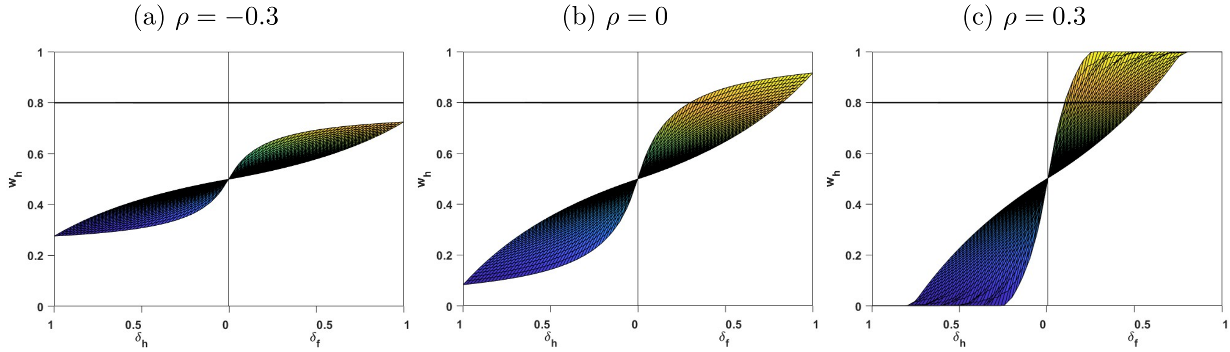
In Figure 1, we illustrate the joint effect of ambiguity and correlation on the optimal allocation among two assets with identical excess return means and standard deviations and scaled ambiguity parameters from 0 to 1. Panels A-C display the allocations to home when $\rho = -0.3, 0, 0.3$, respectively. To the right of the origin, we display the allocations with increasing δ_f and $\delta_h < \delta_f$; to the left, we display increasing δ_h and $\delta_h > \delta_f$; at the origin $\delta_h = \delta_f$. Since we plot for a range of δ_h for a given δ_f, and vice versa, the plots display more than one curve, converging to a unique point when either ambiguity takes the extreme value of 1. The unbiased allocation of 0.5 is obtained with identical ambiguities. To the right, as δ_f increases with higher foreign ambiguty than home, the portfolio tilts towards the home. The reverse is true to the left. However, the bias depends on the correlation. For negative correlation, the home allocation goes up to 0.75; with zero correlation, it goes up to 0.85; for positive, it reaches 1 for small values of foreign ambiguity. The horizontal line shows an empirically observed home allocation of 0.80, about the global average. The optimal allocations do not cross the observed one in panel A, and ambiguity can not explain the home bias. In the other panels, there are multiple crossings, and the model can explain the puzzle for small values of foreign ambiguity.

From Corollary 2.1, we obtain portfolio weights to study how ambiguity can tilt them towards or away from a benchmark to inform the home bias puzzle. In Appendix Corollary C.1, we derive the sensitivity of diversification to ambiguity, showing that ambiguity is a determinant of under-diversification independently of a benchmark.

Ambiguity sets in the two-asset case do not have correlated returns; we add them next.

Figure 1: Allocation among two correlated assets with heterogenous ambiguity

This figure illustrates the optimal asset allocation using the worst-case model (6) for two assets with identical monthly excess return means (0.6%) and standard deviations (4.3%), scaled ambiguity parameters δ_h and δ_f from 0 to 1, and correlation $\rho = -0.3, 0, 0.3$ in panels A-C, respectively. To the right of the origin, we display the home allocation with increasing ambiguity δ_f and $\delta_h < \delta_f$; to the left, we display the allocation for increasing δ_h and $\delta_h > \delta_f$; at the origin $\delta_h = \delta_f$. The horizontal line indicates the empirically observed global average home allocation.



2.1.3 Correlated returns ambiguity

Optimal portfolios should not necessarily be tilted toward less ambiguous assets when considering risks and rewards. For instance, negatively correlated ambiguous assets may be preferable to positively correlated unambiguous ones. Our model nests alternative possibilities, with allocations that may tilt towards or away from less ambiguous assets depending on the correlations. Such allocations may be optimal and unbiased, depending on the asset characteristics.

We illustrate that our model with ellipsoidal ambiguity sets can generate solutions where higher foreign ambiguity does not necessarily induce home bias and that home bias is possible with higher home ambiguity. We also show that models without correlated returns ambiguity generate portfolios where relative ambiguity between home and foreign ($m = \delta_h/\delta_f$) less than one always induces bias. In section 5, we go further to show that models without correlated returns ambiguity do not explain the observed bias.

We consider three ambiguous assets, one home and two foreign, with identical expected excess monthly returns and standard deviations as in the two-asset problem, a correlation of 0.4 among home and foreign, and a varying correlation between the foreign assets in the ellipsoid definition. Ambiguities are scaled from 0 to 1.

We display the optimal home allocation in Figure 2, panels A-C, for foreign correlations -0.6, 0, or 0.6, respectively. In panel A, we observe a total allocation to the negatively correlated foreign assets irrespective of the relative ambiguity between home and foreign. In panel B, we observe that it is possible to have zero home allocations even if foreign ambiguity is higher than the home; see, e.g., point P with zero home allocation with

$\delta_f = 1$ and $\delta_h = 0.75$. In panel C with positively correlated foreign assets, we observe positive home allocation even for higher home ambiguity; see point P with positive home allocation with $\delta_f = 0$ and $\delta_h = 0.25$. The model with correlated returns ambiguity sets is unbiased and lets the data speak.

[Insert Figure 2 Near Here]

We solve the same problem using a worst-case MtC model with interval ambiguity sets (subsection 5.1) and a worst-case mean-variance interval ambiguity model (Boyle et al., 2012). The results are similar with both models, and we display the latter in panels D-I.[7] Panels D-F show the home allocation for varying foreign ambiguity and $\delta_h = 1$, and panels G-I for $\delta_h = 0.75$. The home allocation is increasing with the correlation, like in Boyle et al. (2012). Comparing D with A, we observe that the interval model gives identical results to the ellipsoidal model. However, in G, we observe total allocation to home when home ambiguity is lower than the foreign, even if the foreign assets are negatively correlated, whereas, in A, the home allocation remains zero for any δ's. For zero correlation, we observe (panel E) that the home allocation is positive 0.15 when all three assets are entirely ambiguous, increasing to 1 in H. In contrast, B shows home allocations 0 for the same δ's. For positive correlation, we observe (panel F) a home allocation of 0.40 when the assets are entirely ambiguous, increasing to 1 in I. In contrast, C shows lower home allocations of 0 and 0.75 for the same δ's.

In conclusion, interval ambiguity models induce home bias whenever foreign ambiguity is higher than the home. Depending on the correlations, an ellipsoidal ambiguity model may or may not be home-biased.

2.1.4 Homogeneous ambiguity set

We drop the home asset and reduce the model to one where investors select among assets with homogenous ambiguity. We run simulations for two assets with an ellipsoid of varying size and record the optimal portfolio diversification. We draw returns from a bivariate t-distribution, with degrees of freedom 11, monthly mean 0.6%, and standard deviation 8.1%. We run the model for increasing δ a thousand times to obtain optimal portfolios and repeat the simulation for correlations in the range -0.6 to 0.6.

We compute the commonly used diversification measure $\text{Div}_1 = 1 - w^\top w$, and the measure of Mitton and Vorkink (2007) accounting for the correlations

$$\text{Div}_2 = 1 - \big(w^\top w + (1 - w^\top w)\bar{\rho}\big), \tag{12}$$

[7] We display the results of the model published in the literature, which we implemented with no short sales, and risk aversion of two following the authors' suggestion.

where $\bar{\rho} = \frac{\sum_{i,j=1}^{n} w_i w_j \rho_{ij}}{\sum_{i,j=1}^{n} w_i w_j}$ and ρ_{ij} denotes correlations.

In Figure 3, we display the averages from our simulations for increasing scaled δ until the diversification flattens out. The curves shift towards lower diversification with increasing correlation, as expected. Importantly, we also observe that lower ambiguity comes with less diversified portfolios. This observation aligns with Bianchi and Tallon (2019); Dimmock et al. (2016a) that ambiguity-averse investors hold less diversified portfolios. However, our model links diversification to perceived ambiguity and can be tested using market estimates of perceived ambiguity instead of investor ambiguity tastes.

[Insert Figure 3 Near Here]

With heterogeneous ambiguities in two assets, we showed that increasing ambiguity in one asset tilts the allocation towards the other and reduces diversification. This is consistent with Figure 3, where we observe that the less diversified portfolios are less ambiguous, suggesting that households prefer the less ambiguous assets. Put another way, between heterogeneous assets, the model shifts the allocation to the less ambiguous one and is less diversified. Among homogeneous assets, the model seeks the least ambiguous subset, resulting in less diversified portfolios.

2.2 Model with continuous ambiguity aversion

To solve the general model with continuous ambiguity aversion, we need a tractable formulation of (5) for $\lambda = 0$. To solve the best-case inner problem, we need a tight lower bound for CVaR of portfolio excess return under ambiguity in distribution, akin to the upper bound of Scarf (1958) leading to the results of Chen et al. (2011); Lotfi and Zenios (2018) we used to solve the worst-case problem. We can obtain such a bound from the fundamental minimization formula of CVaR (Theorem C.1) and Jensen's inequality. This lower bound is a piece-wise linear function of portfolio mean excess return and the variable γ of the fundamental minimization problem (i.e., Value-at-Risk, VaR) and is unbounded unless we impose a mild assumption of distributions with finite VaR. We assume a multivariate Student's t-distribution that captures the stylized facts of returns, with degrees of freedom ν, mean $\bar{r}$, and covariance matrix $\hat{\Sigma}$. Using a result from Kamdem (2005), we obtain CVaR analytically to derive a second-order cone program. The model for continuous ambiguity aversion under a t-distribution is given in the following theorem.

Theorem 2.4 (Second-order cone program with continuous ambiguity aversion)**.** *Model (5)* *is cast as:* s.t.

$$\max_{(w'_h, w'_f)\in\mathbb{R}^n_+} \quad (\hat{r}_h - r_f)w'_h + (\hat{r}_f - r_f e)^\top w'_f - (2\lambda - 1)\left(\delta_h \hat{\sigma}_h w'_h + \delta_f \sqrt{w'^\top_f \hat{\Sigma}_f w'_f}\right) \quad (13)$$

$$\begin{aligned} \text{s.t.} \quad & -(\hat{r}_h - r_f)w'_h - (\hat{r}_f - r_f e)^\top w'_f + (2\lambda - 1)\left(\delta_h \hat{\sigma}_h w'_h + \delta_f \sqrt{w'^\top_f \hat{\Sigma}_f w'_f}\right) \\ & + es_{\nu,\alpha}\sqrt{w'^2_h \hat{\sigma}^2_h + 2w'_h \hat{\sigma}^\top_{hf} w'_f + w'^\top_f \hat{\Sigma}_f w'_f} \leq 1 \\ & w'_h + e^\top w'_f > 0. \end{aligned}$$

where $es_{\nu,\alpha}$ *is a parameter obtained from the degrees of freedom of the t-distribution of portfolio returns and the confidence level (Kamdem, 2005, Theorem 4.2). From the optimal solution* $w'^\star_h$ *and* $w'^\star_f$ *we obtain the solution to (5) as* $w^\star_h = \frac{1}{e\top(w'^\star_h + w'^\star_f)} w'^\star_h$ *and* $w^\star_f = \frac{1}{e\top(w'^\star_h + w'^\star_f)} w'^\star_f$. (See Appendix Appendix D.2 for the proof.)

3 Equity home bias

We put the models to the data to explain the equity home bias puzzle. We discuss the data, construct ambiguity sets with two methods, and use the models to select internationally diversified portfolios. We first use the worst-case ambiguity aversion model to examine whether perceived ambiguity (beliefs) explains the observed home bias. We then use continuous ambiguity aversion to infer the ambiguity aversion (tastes) that explains the bias. We solve the second-order cone programs with CVX (Grant and Boyd, 2014), using MATLAB 9.13 on a Core i7 CPU, 2.11GHz processor, and 16 GB of RAM.

3.1 Data

We use the equity indices for 21 developed and 19 emerging markets from the MSCI classification, comparable to the most extensive sample of this literature by Mishra (2015).[8]

3.1.1 Equity holdings

We calculate portfolio weights for the actual equity holdings based on the foreign portfolio assets and liabilities reported in the IMF Coordinated Portfolio Investment Survey (CPIS) database. This is the standard source of data in puzzle studies (Lane and Milesi-Ferretti, 2008), with holdings reported annually in USD for the period 2001-2018.

[8]Our developed markets sample are Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Hong Kong, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, UK, USA. The emerging markets are Brazil, Chile, China, Colombia, Czechia, Egypt, Hungary, India, Israel, Korea, Malaysia, Mexico, Peru, Philippines, Poland, Russia, South Africa, Thailand, Turkey.

3.1.2 Market capitalization

Market capitalization in USD is from the World Federation of Exchanges database.[9] For Italy and Finland, we complete the missing values using market capitalization in EUR and the foreign exchange rates from the European Central Bank. For Denmark and Sweden, we completed using market capitalization data based on NASDAQ OMX indices (in local currency) and foreign exchange rates from Thomson Reuters. A few remaining missing values are filled in from the World Bank database.[10]

3.1.3 Home bias index

We compute the *home bias index* (HBI) of investors in country i as the discrepancy of the actual holdings a_i from the holdings w_i implied from market capitalization (Mishra, 2015). Let EQ_i denote home holdings, MC_i stock market capitalization, and TEQ_i total foreign and domestic equity holdings, i.e., the difference between the country's market capitalization and foreign equity liabilities. Hence, $a_i = \mathrm{EQ}_i/\mathrm{TEQ}_i$. Under the ICAPM, the optimal allocation is each country's market is $w_i = \mathrm{MC}_i/\sum_{j\in \text{all countries}}\mathrm{MC}_j$, with

$$\mathrm{HBI}_i = \frac{a_i - w_i}{1 - w_i}. \tag{14}$$

An index value of 1 signifies complete home bias, and 0 signifies optimally diversified portfolios. In Table A.1, we report the average annual equity home bias index from the IMF CPIS database for all countries in our sample. We report biases with respect to the ICAPM and the minimum variance portfolios without short sales. The two estimates are very close and align with Mishra (2015). The average bias is 0.70 for developed and 0.95 for emerging markets, with standard deviations of 0.14 and 0.06, respectively. Home bias is high despite increasing market integration; see Figure B.1 for the temporal change for developed and emerging markets until 2018, Coeurdacier and Rey (2013) for different regions until 2008, and Ahearne, Griever, and Warnock (2004) for the US until 2000.

3.1.4 Returns

We use the MSCI Investable indices market returns to avoid positive biases when ignoring investability frictions, such as illiquidity risk and index replicability. Data are from Datastream for the period January 1999–December 2019, for 252 monthly observations. For investors in each country, we calculate end-of-month index prices by multiplying the prices in USD by the corresponding contemporaneous FX spot rate. Rsk-free rates are from the sources of Table A.3, obtained from Datastream, except for the Euro area and the US, which are obtained from Refinitiv and Kenneth French's website, respectively.[11]

[9] https://focus.world-exchanges.org/articles/market-capitalisation-q3-2023

[10] https://data.worldbank.org/indicator/CM.MKT.LCAP.CD

[11] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html#Developed

In Table A.2, we give descriptive statistics of excess returns. Most country indices are negatively skewed, justifying a performance ratio without the normality assumption. We also report the ambiguity parameters estimated using two methods, as explained below, for home and the foreign markets corresponding to each home.

3.2 Measuring perceived ambiguity

There has yet to be a consensus on a methodology for constructing ambiguity sets (Bianchi and Tallon, 2019). A typical approach is to use confidence intervals of the statistical estimator of the means (Boyle et al., 2012; Maccheroni et al., 2013; Peijnenburg, 2018; Uppal and Wang, 2003). Aït-Sahalia, Matthys, Osambela, and Sircar (2021) proxy ambiguity using the economic policy uncertainty (EPU) index of Baker, Bloom, and Davis (2016).[12] We use both of these fundamentally different approaches.

First, we estimate the covariance matrix from the time series of historical returns over the whole sample and use a 60-month rolling window to obtain 193 estimates of the mean returns. We then follow Lotfi and Zenios (2018) to construct the smallest ellipsoid centered at the minimum sum of distances from all estimates. We call this *data-based* ambiguity set. Its size $\delta_{d,h}$ for each home country is given in Table A.2, together with $\delta_{d,f}$ of the ellipsoids for the foreign returns of each country.

Second, we generate *EPU-based* ambiguity sets following Aït-Sahalia et al. (2021).[13] We normalize the country EPU values such that their mean across time and country equals one, and for each country i, obtain the expected return confidence interval as

$$\hat{r}_i - \mathrm{EPU}_i \frac{\Psi^{-1}(\beta)\hat{\sigma}_i}{\sqrt{T}} \leq \bar{r}_i \leq \hat{r}_i + \mathrm{EPU}_i \frac{\Psi^{-1}(\beta)\hat{\sigma}_i}{\sqrt{T}}. \tag{15}$$

EPU_i is the mean value of the normalized EPU index of country i over time, Ψ is the normal cumulative distribution, and β is the confidence level set at 0.99. $\hat{r}$ and $\hat{\sigma}$ are the mean and standard deviation of returns estimated from the whole sample. We report the ambiguity set size for each home country $\delta_{E,h}$ in Table A.2. For the foreign $\delta_{E,f}$, we use simulation to generate an ellipsoid within the $(n-1)$-dimensional box specified by the intervals from (15). We generate random observations uniformly within the box and construct the smallest ellipsoid as in the data-based method.

The *relative ambiguity* between home and foreign has a mean value of 0.30 for data-based and 0.11 for EPU-based, with the foreign ambiguity always higher than the home.

[12] Proxies of ambiguity by the volatility of volatility (Epstein and Ji, 2013) or errors in volatility predictions (Dlugosch and Wang, 2022) are available only for two markets (VVIX, V-VSTOX).

[13] EPU indices are available for Australia, Belgium, Brazil, Canada, Chile, China, Colombia, Denmark, France, Germany, Greece, Hong Kong, India, Italy, Japan, Korea, Mexico, Netherlands, Russia, Spain, Sweden, UK, USA. They are obtained from `http://www.policyuncertainty.com`, accessed May 2022.

δ is not a function of the ellipsoid's dimension, and ambiguity does not mechanically increase with more countries. As a counter-example, taking Greece as the home and Finland and France as the foreign, we obtain $\delta_{d,h} = 4.17$, higher than $\delta_{d,f} = 3.16$.[14]

3.3 Perceived ambiguity channel

We now put the worst-case ambiguity aversion model to the data of developed countries to examine whether beliefs explain the observed home bias. We select optimal portfolios for varying levels of foreign perceived ambiguity δ_f with home ambiguity set size $\delta_h = m\delta_f$. We test for $m = 0.30$, the empirically observed average relative ambiguity for all developed countries in the sample, for $m = 0$, assuming unambiguous home returns and an intermediate $m = 0.20$. For $m = 0.30$, the home perceived ambiguity is as obtained from the data, and for lower values, the investor perceives lower ambiguity for home. We compute the ICAPM home bias of the optimal portfolios and display the results in Figure 4 for countries with a large HBI in the range 0.71–0.86 (Australia, Germany, Japan, USA) and for Norway with HBI 0.38 much below the average of developed markets. The horizontal line displays the observed HBI from the CPIS data.

[Insert Figure 4 Near Here]

We observe that the bias increases with ambiguity in the foreign markets (δ_f) and decreases with the relative ambiguity of home (m). This is in line with existing literature that ambiguity induces bias. What is remarkable is that the model generates home allocations that match the observed home bias in all cases. The model-generated curves cross the observed bias line for perceived ambiguity values within the data- and EPU-based estimates. Similar results for the other developed countries are displayed in Figure B.2, and this finding holds for the minimum variance home bias index from Table A.1.

We estimate the value of foreign perceived ambiguity for each country for which the model-generated home allocation matches the observed one. We obtain the optimal portfolio for increasing δ_f to find the *crossover* δ_c when the optimal portfolio matches the observed home allocation. The home relative ambiguity m is from Table 1, and we also consider home relative ambiguity as the average over all countries $\bar{m}$ for a robustness test. We report the crossover values $\delta_c(m)$ and $\delta_c(\bar{m})$ in Table 1 for data-based (panel A) and EPU-based (panel B) ambiguity estimates. The crossover foreign ambiguity for which the observed home bias is optimal is well within the data- and EPU-based estimates.

[Insert Table 1 Near Here]

In conclusion, the observed equity home allocations are optimal for the worst-case

[14] The optimal portfolio for this example is not home-biased, but its performance MtC ratio of 0.041 is much lower than 0.089 when diversifying among all 40 countries with significant home bias.

ambiguity-averse investors with concave, non-decreasing utility functions once we account for the international markets' perceived ambiguity. Ambiguity beliefs explain the puzzle.

One potential concern is that our perceived ambiguity estimates are unreasonably high; this concern is alleviated by obtaining estimates with two fundamentally different and well-accepted methods. Another concern could be that the market-estimated ambiguities suggest even higher allocations to home; this is a result of the worst-case ambiguity aversion model. Both concerns are addressed with the continuous ambiguity aversion model. Taking the perceived ambiguity estimates as given, the general model matches precisely the observed home allocations with mild ambiguity aversion. Without extreme ambiguity aversion, we do not get excessively high home allocations, addressing the second concern. Furthermore, in three cases, the estimated ambiguity aversion is close to experimental estimates from population samples. This validates the perceived ambiguity estimates we used to infer the ambiguity aversion, addressing the first concern.

3.4 Ambiguity aversion channel

We use the general model to infer the ambiguity aversion for which the optimal home allocations match the observed ones under the ambiguity estimates of Table A.2. The t-distribution degrees of freedom are set to the average of developed countries, $\nu = 11$.[15]

We list in Table 2 the ambiguity aversion parameter that explains the puzzle for each country. The average is 0.60 and 0.64 for data- and EPU-based ambiguities, respectively, with a standard deviation of 0.06. These values are higher than the ambiguity neutral 0.5 (p-value 0.01) and are statistically indistinguishable (p-value 0.11). Hence, mild ambiguity aversion explains the equity home bias of all developed countries for perceived ambiguity estimated with two different methods.

[Insert Table 2 Near Here]

The results of this section complement our finding above that under worst-case ambiguity-aversion, perceived ambiguity well within the market estimates generates the observed home bias. We now find that ambiguity aversion explains the observed bias under the market estimates of perceived ambiguity. Both beliefs and tastes explain the puzzle.

3.5 Consistency of model estimates

One potential concern is that as the markets' perceived ambiguity changes, the inferred ambiguity aversion can change mechanically to match the observed home allocation and does not reflect the tastes of market participants. Although we obtained consistent ambiguity aversion estimates from two fundamentally different ambiguity sets, we go further

[15]The results are robust to values of ν in the range 3 to 100 observed for developed countries.

to show that our estimates align with experimental observations and theoretical expectations.

First, our estimates closely match experimental evidence from the literature. On a sample of US households, Dimmock et al. (2016a) found an average ambiguity aversion of 0.52 compared to our estimates of 0.59 (data-based) or 0.62 (EPU-based). Dimmock et al. (2016b) find an average ambiguty aversion of 0.56 on a sample of Dutch households, with our corresponding estimates 0.59 or 0.63.[16] The consistency with experimental evidence alleviates concerns that the inferred values are arbitrary.

Second, we run a test exploiting the fact that the home bias index has been declining over time; see Ahearne et al. (2004); Coeurdacier and Rey (2013) and Figure B.1. We apply the model to three equally sized periods. We could potentially explain the declining home bias through reduced perceived ambiguity, lower ambiguity aversion, or a combination. Perceived ambiguity may decline over time due to elimination of factors that create ambiguity, e.g., deregulation (Cooper et al., 2012) and increasing market integration (Baele et al., 2007), transparency (Gelos and Wei, 2005), and lower information asymmetries (Andrade and Chhaochharia, 2010). However, ambiguity tastes are not expected to change with time, much like constant relative risk aversion is a standard assumption in macroeconomic and asset pricing models (Hansen and Singleton, 1982; Mehra and Prescott, 1985; Vissing-Jørgensen and Attanasio, 2003), and has been documented using panel data (Chiappori and Paiella, 2011). Hence, if we explain the puzzle for different periods with an invariant ambiguity aversion, then our model results would be consistent with theoretical expectations.

We consider the periods 1999-2005, 2006-2012, and 2013-2019, and for each, we use a 24-month rolling window to estimate the data-based perceived ambiguity parameter. We then test both channels as in sections 3.3 and 3.4, for each period. We report summary results for each period and the full sample in Table 3. In Panel A, we give the average home, foreign, and relative ambiguity estimates, the cross-over points, and the ambiguity aversion parameters, and in Panel B, we report the p-values for pairwise differences.

We observe an almost monotonic decrease of the home, foreign, and relative ambiguity from the earlier to the later periods. The crossover points are within the perceived ambiguity estimates and are statistically indistinguishable, corroborating the explanation through ambiguity beliefs. Importantly, the ambiguity aversion that explains the puzzle remains invariant across the three periods and the full sample (p-values 0.12 to 0.94).

[Insert Table 3 Near Here]

[16] These two references use different ranges for ambiguity aversion, and for the comparison, we transform them linearly into the 0 to 1 range. A statistical test on their estimates found them significantly different than the ambiguity-neutral value (p-value < 0.01).

4 Household portfolio under-diversification

We verify the model prediction that lower homogeneous ambiguity comes with less diversified portfolios using US household data.

4.1 Data

We use the Barber and Odean (2000) database of household monthly portfolio holdings from a major US discount brokerage house. We aggregate households with multiple entries to a single household portfolio (Mitton and Vorkink, 2007) and exclude portfolios with no equity or short positions during the sample period. We consider the three most recent years in the database —January 1996, 1995,1994— for 23,096, 27,248, and 34,060 households, respectively. To estimate portfolio perceived ambiguity, we use the data-based approach and the degree of disagreement among financial analysts suggested by Anderson, Ghysels, and Juergens (2009).

From the original database, we construct two samples with complete data for estimating ambiguity sets with both methods. We first select portfolios of securities available in the Center for Research in Security Prices (CRSP) data and obtain monthly returns over the preceding ten years. Further, we restrict our sample to portfolios of securities in the Institutional Brokers Estimate System (I/B/E/S) with a monthly standard deviation of financial analysts' next-quarter earnings forecasts over the last ten years. From the first sample, we estimate the data-based ambiguity parameter of each portfolio and drop the 2% largest and smallest outliers to obtain 18,059, 22,321, and 28,539 portfolios for the three dates. From the second sample, we estimate the analyst-based ambiguities and drop the 2% outliers for 8,397, 10,672, and 13,476 portfolios, respectively. In Table A.4, we give statistics of the number of portfolios of different sizes in our samples, with similar characteristics to the original data and most households holding very few securities. In Table A.5 we give statistics for portfolio diversification and returns.

4.2 Measuring perceived ambiguity

We first use a 24-month rolling window over the preceding ten years to obtain 97 estimates of the mean returns and the covariance matrix over the ten years to construct the data-based ellipsoid as in subsection 3.2.[17]

Alternatively, we use the dispersion of analysts' forecasts for each portfolio security as a proxy for heterogeneous beliefs about expected returns following Anderson et al. (2009). We normalize the standard deviation of analyst earning forecasts (SDAF) over

[17]The ellipsoid is constructed using only those stocks within a portfolio that have at least 24 estimates of mean returns out of the maximum possible of 97.

the preceding ten years such that its mean across time and portfolio securities equals one. For each security i, we obtain the mean return confidence interval as:

$$\hat{r}_i - \text{SDAF}_i \frac{\Psi^{-1}(\beta)\hat{\sigma}_i}{\sqrt{T}} \leq \bar{r}_i \leq \hat{r}_i + \text{SDAF}_i \frac{\Psi^{-1}(\beta)\hat{\sigma}_i}{\sqrt{T}}. \tag{16}$$

SDAF_i is the mean value of the normalized SDAF, Ψ is the normal cumulative distribution, and β is the confidence level set at 0.99. $\hat{r}$ and $\hat{\sigma}$ are the mean and standard deviation of returns estimated over the preceding ten years. We construct *analyst-based* ambiguity sets from these estimates as with the EPU-based sets in section 3.2.

4.3 Prediction verification

We sort the household portfolios into deciles by diversification and construct each portfolio's data- and analyst-based ellipsoidal ambiguity sets. In Table 4, we report the average diversification Div_1 (panel A) and Div_2 (panel B) together with the average perceived ambiguity of the portfolios within each decile obtained with the data- or analyst-based methods (sub-panels i and ii, respectively). We observe monotonically increasing ambiguity with diversification, verifying the model's prediction. We also report return statistics, with the portfolio skewness exhibiting a negative relation with diversification as uncovered by Mitton and Vorkink (2007).

[Insert Table 4 Near Here]

We run cross-sectional regressions on diversification with controls for $k = 1, 2$:

$$\text{Div}_{k,i} = \alpha + \beta \log(\delta_i) + \theta_1 \,\text{Sharpe}_\text{i} + \theta_2 \,\text{Skew}_i + \theta_3 \log(\text{PrtfValue}_i) + \epsilon_i. \tag{17}$$

The logarithm accounts for non-linearities, and δ_i is the ambiguity estimate of the ith portfolio. We control for portfolio Sharpe ratio following standard finance theory, skewness (Skew) following Mitton and Vorkink (2007), and portfolio value (PrtfValue) following Goetzmann and Kumar (2008). We run regressions on data- and analyst-based ambiguity estimates for January 1996, 1995 and 1994.

We report the results in Table 5 with Div_1 (panel A) and Div_2 (panel B), with both data- and analyst-based ambiguity estimates (sub-panels i and ii, respectively). The ambiguity coefficient is statistically significant in all specifications, comparable in magnitude to Sharpe ratio and an order of magnitude larger than skewness, establishing ambiguity as a determinant of household (under)diversification. This finding suggests that households choose familiar assets (Boyle et al., 2012; Cao et al., 2011). The Sharpe ratio and skewness coefficients are positive and negative, respectively, in line with standard finance literature and Mitton and Vorkink (2007).

[Insert Table 5 Near Here]

5 Further tests

We perform additional tests to (i) show that without correlated returns ambiguity, we can not explain the equity home bias puzzle, (ii) show that the model explains the puzzle for emerging markets, and (iii) alleviate potential data mining concerns.

5.1 Interval ambiguity

We develop the worst-case model with interval ambiguity sets in Appendix D.3 and show on a three-country example that it does not generate the observed home bias for reasonable ambiguity parameters. Specifically, we consider an ambiguity set specified by a maximum interval and control its magnitude using a shrinkage factor from 0 to 1; 0 shrinks the ambiguity interval to its mean, and one is for maximum ambiguity.

In Figure 5 (panel A), we display the maximum interval ambiguity sets for Japan, USA, and Germany. Japan is less ambiguous than the other two countries, so a model with ambiguity aversion tilts the portfolio towards Japan.[18] However, we see in panel B that for the model allocation to match the observed allocation, we must expand the ambiguity set of the foreign assets (i.e., use a shrinkage factor greater than one) and shrink the ambiguity set for Japan. Assuming that the Japanese ambiguity is half what we observe in the data, we obtain the home bias for higher foreign ambiguities than observed in the data (shrinkage factor above 1.1). If the perceived home ambiguity is as we observe in the data, we must expand the foreign ambiguity by a factor of more than two. The model with interval ambiguity sets needs fine-tuning with arbitrary parameter values to match the observed allocations, failing the test of Cooper et al. (2012). We obtained similar results with the model of Boyle et al. (2012); the interval ambiguity of the foreign needs to be increased by a factor of three for the home allocation to match the observed one.

[Insert Figure 5 Near Here]

We also test our continuous ambiguity aversion model with intervals.[19] This model gives an average home bias index of about 0.4 for ambiguity aversion parameters ranging from 0.5 to 1. The maximum bias (0.6) is well below the observed 0.8.

[18]This example is not contrived, and our models with ellipsoidal ambiguity sets explain this bias as well with perceived foreign ambiguity well within the data- and EPU-based ambiguity estimates and for reasonable home ambiguity aversion parameter.

[19]Replacing ellipsoids by intervals in the continuous ambiguity aversion model follows easily from the steps deriving the interval version of the ellipsoidal worst-case model in Appendix D.3.

Under interval ambiguity, neither a mean-variance model nor our model with higher-order moments or a continuous ambiguity aversion model can explain the puzzle.

5.2 Equity home bias for emerging markets

Capital controls and other frictions play a significant role in inducing home bias in emerging markets (Cooper et al., 2012), but we find that correlated returns ambiguity explains the puzzle. Our main results of Table 1 for beliefs and Table 2 for tastes hold for emerging markets. We show consolidated results in Table 6, reporting the cross-over ambiguities, the data- and EPU-based perceived ambiguities, and ambiguity aversion.

Ambiguity beliefs explain the bias for all countries with both data- and EPU-based ambiguity estimates, except for Russia and Colombia, where the explanation only holds with the EPU-based estimates. The average ambiguity aversion explaining the bias is 0.58 (data-based) or 0.61 (EPU-based), close to the corresponding values of 0.60 and 0.64 for developed markets (p-value 0.10). Potamites and Zhang (2012) estimate an average ambiguity aversion of 0.68 on a Chinese population sample, in line with our estimates of 0.58 (data-based) or 0.63 (EPU-based).

[Insert Table 6 Near Here]

5.3 Potential data mining

We further test the validity of the household under-diversification results, estimating the ambiguity parameters using data after the portfolio dates to test for ambiguity expectations following Mitton and Vorkink (2007). We repeat the portfolio sorts and regression (17) for January 1994, 1995, and 1996, obtaining ambiguity estimates for the ten years following the portfolio date. We also performed the tests with observed and expected ambiguity estimates on portfolios formed in January 1991, 1992, and 1993. The results (not reported) are in line with Tables 4 and 5.

6 Conclusion

We developed portfolio selection models with heterogeneous ellipsoidal ambiguity sets of correlated returns and a performance ratio without a normality assumption and explained the equity home puzzle through the ambiguity channel. First, we develop a parsimonious model without risk or ambiguity aversion parameters and show it to be consistent second-order stochastic dominance. We generalize the model for continuous ambiguity aversion. These models account for perceived ambiguity (beliefs) and ambiguity aversion (tastes) as potential puzzle explanations.

Taking the models to the data, we obtain optimal allocations matching those of international investors in 21 developed and 19 emerging markets. This is shown both for worst-case ambiguity aversion under perceived ambiguity well within the market-estimated ambiguity sets or for mild ambiguity aversions given the market-estimated ambiguity sets. The global average ambiguity aversion is about 0.6, statistically different from the ambiguity-neutral 0.5. Our estimates of ambiguity aversion from the observed asset allocations for a large sample of countries closely match the scant experimental evidence from US, Dutch, and Chinese population samples. Our findings are robust to two fundamentally different methods for measuring ambiguity.

The worst-case model applied to a homogeneous ambiguity set predicts lower ambiguity with less diversified portfolios. We verify this on a large dataset of US household portfolios, and by running a regression with controls, we document ambiguity as a significant determinant of household (under)diversification.

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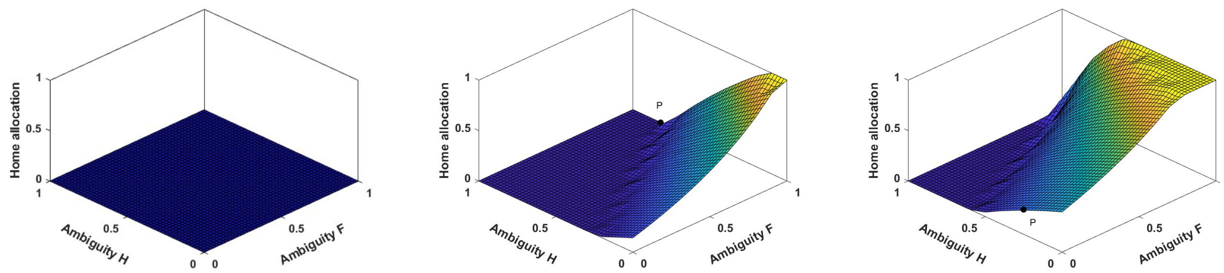
Main Figures and Tables

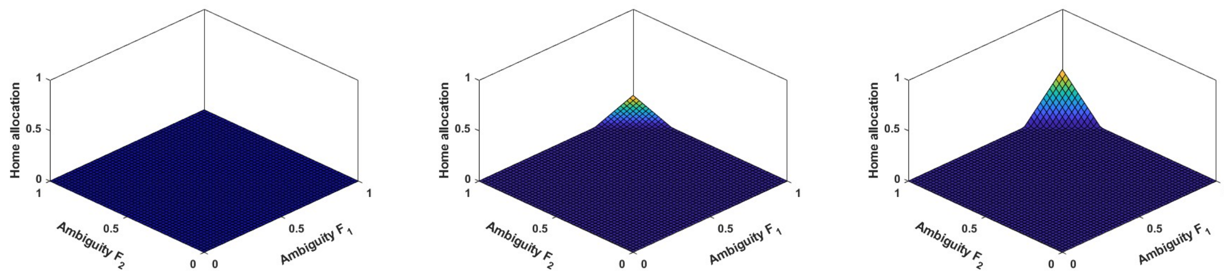
Figure 2: Optimal allocation with ellipsoidal and interval ambiguities

This figure illustrates the home allocation for the case of three ambiguous correlated assets as a function of the ambiguity parameters when means and standard deviations of all assets are equal and correlation among the foreign assets ρ_f of -0.6, 0, and 0.6. Ambiguity parameters are scaled in the range 0 to 1. Panels A-C display home allocations for varying home and foreign ambiguity using model (6) with ellipsoidal ambiguity sets. Panels D-F show the home allocation using interval ambiguity sets according to Boyle et al. (2012) with a home ambiguity of 1 and varying foreign ambiguities. In panels G-I, the home ambiguity is 0.75. The monthly excess return means and standard deviation of the three assets are equal to 0.6% and 4.3%, respectively, and the correlation between home and foreign assets is 0.40.

(a) Ellipsoidal, $\rho_f = -0.6$ (b) Ellipsoidal, $\rho_f = -0$ (c) Ellipsoidal, $\rho_f = 0.6$



(d) Interval, $\delta_h = 1$, $\rho_f = -0.6$ (e) Interval, $\delta_h = 1$, $\rho_f = 0$ (f) Interval, $\delta_h = 1$, $\rho_f = 0.6$



(g) Interval, $\delta_h = 0.75$, $\rho_f = -0.6$ (h) Interval, $\delta_h = 0.75$, $\rho_f = 0$ (i) Interval, $\delta_h = 0.75$, $\rho_f = 0.6$

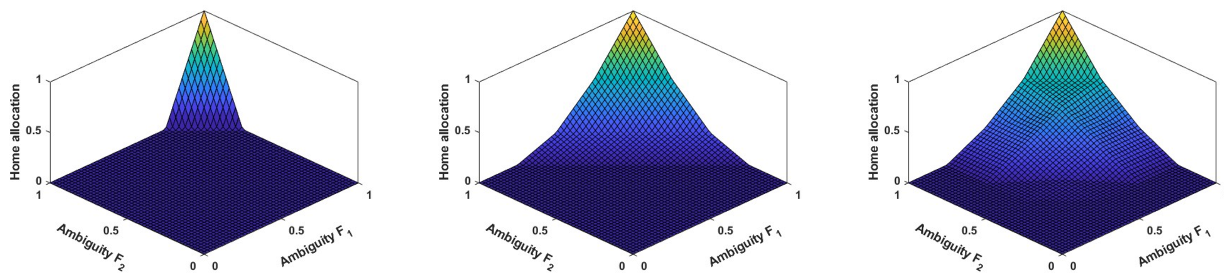
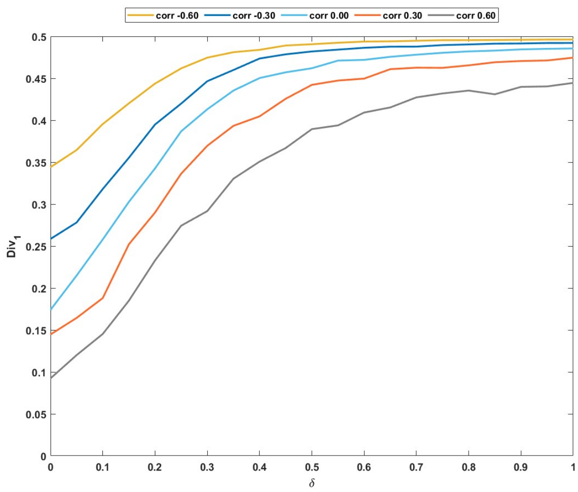


Figure 3: Diversification with a homogenous ambiguity set

This figure illustrates the results of a simulation that estimates diversification measures with a homogenous ambiguity parameter δ for a portfolio of two assets using the worst-case model. Panels A and B show the results for diversification measures Div_1 and Div_2, for correlation coefficients ranging from -0.6 to 0.6. For each value of δ, we display the average diversification measures over 1000 repetitions of the model until the diversification flattens out, and we scale δ from 0 to 1. Returns are generated from a bivariate t-distribution, with degrees of freedom 11, monthly mean 0.6%, and standard deviation 8.1%.

(a) Div_1



(b) Div_2

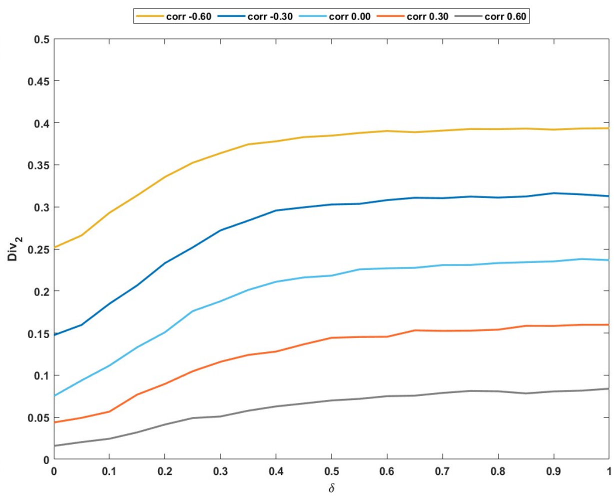
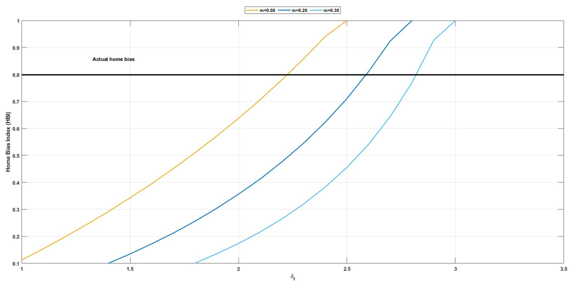


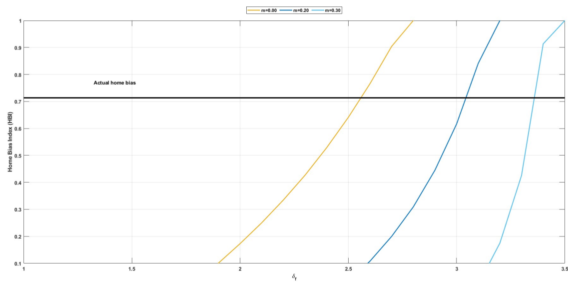
Figure 4: Optimal home allocation and perceived ambiguity

This figure illustrates the home bias index (HBI) of the worst-case model optimal allocations with respect to the market capitalization, as a function of foreign ambiguity (δ_f) for relative ambiguity of home to foreign (m) equal to 0, 0.2, or 0.3. The horizontal line indicates the observed time-average home bias estimated with respect to the ICAPM. Results are displayed for investors in Australia, Japan, Germany, USA, and Norway, with portfolios selected from the 21 developed and 19 emerging markets sample. The sample period spans January 1999 to December 2019.

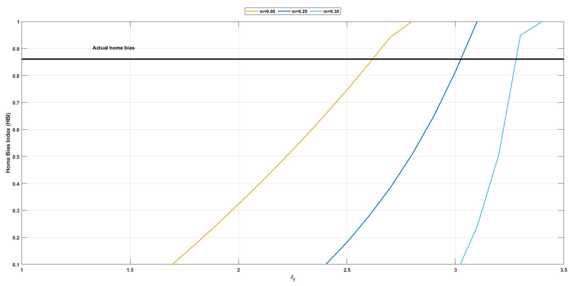
(a) Australia



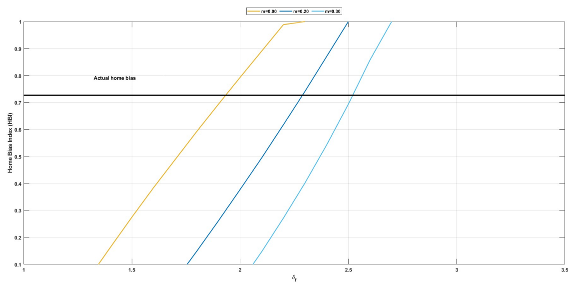
(b) Germany



(c) Japan



(d) USA



(e) Norway

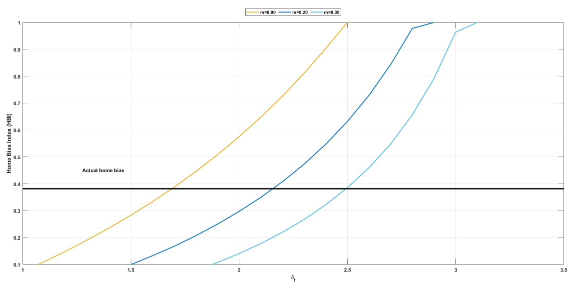


Figure 5: Interval ambiguity does not explain the puzzle

This figure illustrates the optimal allocation to the home market, with varying ambiguity in the home and foreign mean returns, using the worst-case model with interval ambiguity sets on a sample consisting of Japan (home) and Germany and USA (foreign). Panel A reports the ambiguity intervals of mean returns. Panel B illustrates the home allocation as a function of foreign mean returns ambiguity obtained by adjusting the interval of panel A by a shrinkage factor from 0 to 1, and for home ambiguity with shrinkage factor 0, 0.5, or 1. The sample period spans from January 1999 to December 2019.

(a) Interval ambiguity sets

	Germany	Japan	USA
Min	-0.52	-0.35	-0.16
Max	1.81	1.29	1.57
Interval	2.34	1.64	1.73

(b) Home allocations from the model with worst-case interval ambiguity sets

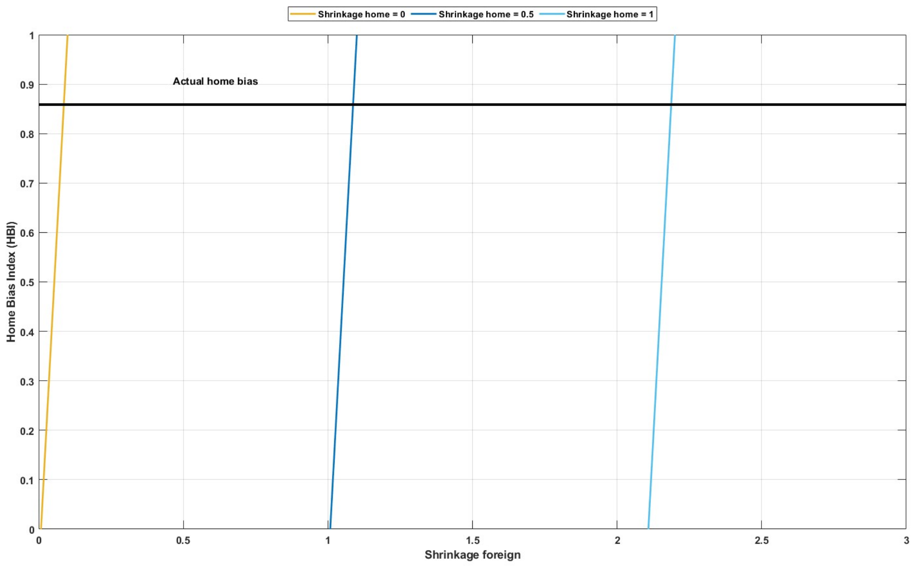


Table 1: Perceived ambiguity and the equity home bias puzzle in developed markets

This table reports the cross-over foreign ambiguity parameter δ_c for which the optimal allocation with the worst-case model has a home bias equal to the observed one. Also reported are the data- ($\delta_{d,f}$) and EPU-based ($\delta_{E,f}$) ambiguity estimates for the set of foreign markets for each country, the corresponding home estimates ($\delta_{d,h}, \delta_{E,h}$), and the relative ambiguity (m). Panel A and B report results with the data- and EPU-based ambiguity estimates, respectively. $\delta_c(m)$ and $\delta_c(\bar{m})$ are the cross-over values for the relative ambiguity of each country (m) or the country average ($\bar{m}$). In panel A, the model selects portfolios from the sample of 21 developed and 19 emerging markets. The model is solved in panel B for the sub-sample of 15 developed and eight emerging markets of countries with EPU ratings. Countries without EPU data have missing entries marked -. The sample period spans January 1999–December 2019.

	(a) Data-based					(b) EPU-based				
Country	$\delta_c(m)$	$\delta_c(\bar{m})$	$\delta_{d,f}$	$\delta_{d,h}$	m	$\delta_c(m)$	$\delta_c(\bar{m})$	$\delta_{E,f}$	$\delta_{E,h}$	m
Australia	2.9	2.8	12.21	3.92	0.32	1.7	1.7	7.07	0.71	0.10
Austria	3.3	3.1	12.46	4.43	0.36	-	-	-	-	-
Belgium	3.3	3.3	12.47	3.84	0.31	2.6	2.6	7.99	0.78	0.10
Canada	3.0	3.0	12.34	3.71	0.30	1.6	1.5	7.66	1.14	0.15
Denmark	1.4	1.5	12.48	2.96	0.24	0.6	0.6	8.75	0.80	0.09
Finland	3.7	5.1	12.47	2.87	0.23	-	-	-	-	-
France	3.3	3.5	12.23	3.13	0.26	2.3	2.1	6.32	1.30	0.21
Germany	3.2	3.3	12.24	3.13	0.26	2.4	2.3	7.06	1.02	0.15
Greece	-	-	12.37	4.17	0.34	5.6	5.8	8.56	0.88	0.10
Hong Kong	2.4	2.5	12.31	3.23	0.26	1.7	1.7	9.16	0.94	0.10
Italy	5.8	5.2	12.45	4.07	0.33	2.9	2.9	7.16	0.80	0.11
Japan	3.6	3.3	12.20	4.29	0.35	2.1	2.2	10.67	0.80	0.08
Netherlands	3.1	3.3	12.41	3.15	0.25	1.9	2.0	7.46	0.68	0.09
New Zealand	1.4	1.3	12.10	4.26	0.35	-	-	-	-	-
Norway	2.7	2.5	12.38	4.35	0.35	-	-	-	-	-
Portugal	6.1	5.4	12.44	4.30	0.35	-	-	-	-	-
Spain	3.6	3.6	12.46	3.71	0.30	2.7	2.7	7.74	0.85	0.11
Sweden	3.2	3.2	12.33	3.41	0.28	1.9	1.9	7.75	0.67	0.09
Switzerland	2.1	2.2	12.31	3.55	0.29	-	-	-	-	-
UK	3.0	3.2	12.46	3.15	0.25	2.2	2.2	9.07	0.88	0.10
USA	2.6	2.5	12.00	3.91	0.33	1.5	1.5	8.96	0.90	0.10
Mean	3.2	3.2	12.34	3.69	0.30	2.2	2.2	8.09	0.88	0.11
StdDev	1.1	1.1	0.13	0.51	0.04	1.1	1.1	1.11	0.17	0.03

Table 2: Ambiguity aversion and the equity home bias puzzle in developed markets

This table reports the ambiguity aversion parameter of the home investor for which the optimal allocations with the continuous ambiguity-aversion model have a home bias equal to the observed one. Perceived ambiguity is obtained using both the data- and EPU-based methods. The model selects portfolios from the sample of 21 developed and 19 emerging markets for the data-based column. For the EPU-based column, the model is solved for the sub-sample of 15 developed and eight emerging markets of countries with EPU ratings. Countries without EPU data have missing entries marked -. The sample period spans January 1999–December 2019.

Country	Data-based	EPU-based
Australia	0.57	0.62
Austria	0.57	-
Belgium	0.58	0.66
Canada	0.58	0.60
Denmark	0.53	0.53
Finland	0.60	-
France	0.61	0.69
Germany	0.59	0.67
Greece	0.84	0.84
Hong Kong	0.58	0.60
Italy	0.66	0.70
Japan	0.68	0.60
Netherlands	0.59	0.63
New Zealand	0.54	-
Norway	0.52	-
Portugal	0.68	-
Spain	0.61	0.68
Sweden	0.54	0.62
Switzerland	0.59	-
UK	0.62	0.62
USA	0.62	0.59
Mean	0.60	0.64
StdDev	0.06	0.06

Table 3: The stability of ambiguity tastes over time

This table reports the average of ambiguity tastes and beliefs for developed markets during different subperiods. In Panel A, we report the average of cross-over foreign ambiguity parameter δ_c and ambiguity aversion parameter λ for which the optimal allocations with the worst-case and continuous ambiguity aversion models, respectively, have home bias equal to the observed one. Also reported are the average of data-based ambiguity estimates for the set of foreign markets ($\delta_{d,f}$), the corresponding home estimate ($\delta_{d,h}$), and the relative ambiguity (m). The $\delta_c(m)$ and $\delta_c(\bar{m})$ are the cross-over values for relative ambiguity for each country (m) or the country average ($\bar{m}$). Panel B reports the p-value of the pairwise differences across subperiods. The model is solved on the sample of 21 developed and 19 emerging markets and for three equal subperiods of the overall 20-year sample period P, namely P1 from 1999 to 2005, P2 from 2006 to 2012, and P3 from 2013 to 2019.

	$\delta_c(m)$	$\delta_c(\bar{m})$	$\delta_{d,f}$	$\delta_{d,h}$	m	λ
	(a) Ambiguity beliefs, tastes, and cross-over					
P1 (1999-2005)	2.7	2.8	13.51	4.44	0.33	0.60
P2 (2006-2012)	3.2	3.2	11.50	4.94	0.43	0.64
P3 (2013-2019)	2.4	2.3	9.97	2.07	0.21	0.62
P (1999-2019)	3.2	3.2	12.34	3.69	0.30	0.60
	(b) p-values					
P1-P2	0.41	0.47	0.00	0.11	0.00	0.12
P1-P3	0.51	0.36	0.00	0.00	0.00	0.47
P2-P3	0.17	0.12	0.00	0.00	0.00	0.55
P1-P	0.30	0.33	0.00	0.00	0.10	0.94
P2-P	0.96	0.97	0.00	0.00	0.00	0.16
P3-P	0.11	0.04	0.00	0.00	0.00	0.52

Table 4: Household portfolio diversification sorts and their ambiguity

This table reports the average diversification and perceived ambiguity, together with return statistics for household portfolios sorted into deciles by diversification measure Div_1 (panel A) and Div_2 (panel B). In sub-panels i and ii, the perceived ambiguity of the portfolio is estimated using the data- and analyst-based methods, respectively. We report the number of portfolios N in each decile, average diversification (Div_1 respectively Div_2), average perceived ambiguity (δ), expected return (Mean), standard deviation (StdDev), and skewness (Skew). We report data for the January 1996, 1995, and 1994 portfolios. Statistics are for monthly returns.

(a) Div_1

	January 1996						January 1995						January 1994					
Decile	N	Div_1	δ	Mean	StdDev	Skew	N	Div_1	δ	Mean	StdDev	Skew	N	Div_1	δ	Mean	StdDev	Skew
(i) Data-based																		
1	6201	0.00	3.63	0.017	0.114	0.266	7675	0.00	3.74	0.016	0.114	0.293	9760	0.00	3.70	0.016	0.114	0.245
2	1350	0.13	4.64	0.019	0.098	0.199	1677	0.13	4.62	0.017	0.100	0.268	2153	0.14	4.63	0.017	0.100	0.200
3	1358	0.35	4.75	0.018	0.087	0.136	1674	0.34	4.90	0.016	0.087	0.132	2157	0.36	4.93	0.017	0.089	0.132
4	1365	0.46	4.76	0.018	0.083	0.140	1689	0.46	4.92	0.016	0.083	0.119	2154	0.46	4.90	0.017	0.084	0.118
5	1355	0.51	4.94	0.017	0.079	0.088	1690	0.50	5.02	0.016	0.080	0.128	2153	0.51	5.10	0.017	0.081	0.100
6	1352	0.60	5.50	0.018	0.074	0.050	1684	0.59	5.62	0.016	0.073	0.023	2152	0.60	5.72	0.017	0.074	0.011
7	1349	0.67	5.74	0.018	0.070	-0.024	1669	0.66	5.86	0.016	0.069	-0.004	2137	0.67	5.89	0.016	0.070	-0.038
8	1336	0.74	6.25	0.018	0.065	-0.100	1654	0.74	6.36	0.016	0.064	-0.063	2141	0.73	6.45	0.016	0.064	-0.126
9	1324	0.81	6.83	0.017	0.058	-0.209	1629	0.80	6.94	0.016	0.057	-0.201	2088	0.80	7.13	0.016	0.059	-0.220
10	1069	0.88	7.95	0.017	0.052	-0.395	1280	0.88	8.03	0.016	0.052	-0.346	1644	0.88	8.22	0.016	0.053	-0.411
(ii) Analyst-based																		
1	4532	0.00	2.62	0.012	0.088	-0.012	5751	0.00	2.63	0.011	0.087	0.013	7279	0.00	2.75	0.012	0.086	-0.046
2	444	0.17	4.53	0.013	0.076	-0.050	567	0.17	4.65	0.012	0.075	-0.042	717	0.18	4.88	0.013	0.076	-0.070
3	448	0.35	4.45	0.013	0.070	-0.096	568	0.36	4.60	0.011	0.069	-0.125	718	0.37	4.68	0.013	0.068	-0.145
4	445	0.44	4.57	0.012	0.066	-0.104	568	0.44	4.70	0.012	0.068	-0.095	717	0.45	4.68	0.013	0.066	-0.165
5	449	0.48	4.39	0.012	0.066	-0.119	569	0.48	4.38	0.011	0.065	-0.137	718	0.48	4.60	0.013	0.064	-0.178
6	448	0.51	4.74	0.012	0.065	-0.101	565	0.51	4.85	0.011	0.065	-0.117	711	0.51	4.99	0.013	0.064	-0.200
7	445	0.60	5.72	0.013	0.058	-0.130	566	0.60	5.77	0.012	0.058	-0.173	705	0.60	6.05	0.013	0.059	-0.245
8	436	0.66	6.11	0.013	0.056	-0.198	557	0.67	6.20	0.012	0.055	-0.213	708	0.66	6.36	0.013	0.055	-0.295
9	430	0.74	7.09	0.012	0.049	-0.300	542	0.74	7.18	0.012	0.051	-0.325	688	0.74	7.36	0.013	0.052	-0.396
10	320	0.82	8.54	0.013	0.046	-0.477	419	0.83	8.85	0.012	0.047	-0.408	515	0.83	8.99	0.014	0.047	-0.522

Table 4: (continued)

(b) Div_2

	January 1996						January 1995						January 1994					
Decile	N	Div_2	δ	Mean	StdDev	Skew	N	Div_2	δ	Mean	StdDev	Skew	N	Div_2	δ	Mean	StdDev	Skew
(i) Data-based																		
1	6201	0.00	3.63	0.017	0.114	0.266	7675	0.00	3.74	0.016	0.114	0.293	9760	0.00	3.70	0.016	0.114	0.245
2	1353	0.02	4.62	0.019	0.097	0.188	1676	0.02	4.62	0.017	0.099	0.251	2150	0.02	4.66	0.017	0.099	0.188
3	1361	0.08	4.77	0.018	0.085	0.076	1682	0.08	4.93	0.016	0.083	0.066	2166	0.08	4.94	0.017	0.084	0.035
4	1364	0.14	4.83	0.018	0.080	0.052	1689	0.14	5.00	0.017	0.080	0.061	2154	0.14	5.00	0.017	0.082	0.052
5	1350	0.19	5.01	0.017	0.079	0.099	1686	0.18	5.13	0.016	0.080	0.103	2148	0.19	5.20	0.016	0.080	0.088
6	1356	0.25	5.42	0.017	0.074	0.045	1673	0.24	5.54	0.016	0.074	0.056	2148	0.24	5.63	0.017	0.076	0.045
7	1361	0.31	5.81	0.018	0.070	-0.013	1675	0.30	5.91	0.016	0.068	-0.023	2145	0.31	5.99	0.016	0.069	-0.063
8	1330	0.38	6.27	0.017	0.065	-0.104	1655	0.37	6.45	0.016	0.064	-0.072	2124	0.37	6.45	0.016	0.066	-0.098
9	1295	0.46	6.93	0.017	0.061	-0.186	1614	0.45	6.89	0.016	0.060	-0.122	2061	0.44	7.14	0.016	0.061	-0.172
10	1088	0.56	7.63	0.017	0.056	-0.239	1296	0.55	7.72	0.015	0.056	-0.232	1683	0.55	7.87	0.016	0.057	-0.269
(ii) Analyst-based																		
1	4532	0.00	2.62	0.012	0.088	-0.012	5751	0.00	2.63	0.011	0.087	0.013	7279	0.00	2.75	0.012	0.086	-0.046
2	445	0.02	4.64	0.013	0.074	-0.049	568	0.02	4.74	0.012	0.075	-0.048	717	0.02	4.94	0.013	0.075	-0.072
3	446	0.07	4.61	0.012	0.069	-0.100	569	0.08	4.82	0.011	0.067	-0.111	716	0.08	4.89	0.013	0.067	-0.154
4	445	0.11	4.49	0.013	0.067	-0.125	567	0.12	4.58	0.012	0.067	-0.160	716	0.11	4.71	0.013	0.067	-0.152
5	449	0.15	4.61	0.012	0.065	-0.141	567	0.15	4.83	0.012	0.066	-0.114	716	0.15	4.84	0.013	0.065	-0.208
6	448	0.18	4.82	0.013	0.064	-0.096	567	0.18	4.78	0.011	0.063	-0.115	706	0.18	5.00	0.012	0.062	-0.195
7	443	0.23	5.48	0.012	0.059	-0.157	564	0.23	5.52	0.011	0.059	-0.167	713	0.22	5.87	0.013	0.059	-0.242
8	439	0.29	6.19	0.012	0.055	-0.162	548	0.28	6.31	0.012	0.056	-0.194	698	0.28	6.39	0.013	0.056	-0.287
9	423	0.36	6.96	0.013	0.052	-0.297	545	0.35	7.01	0.012	0.053	-0.303	681	0.34	7.20	0.013	0.052	-0.381
10	327	0.46	8.19	0.013	0.048	-0.433	426	0.45	8.43	0.011	0.049	-0.423	534	0.44	8.53	0.013	0.050	-0.516

Table 5: Regression of household portfolio diversification on perceived ambiguity

This table reports the coefficient of regression (17) of the diversification measure Div_1 (panel A) and Div_2 (panel B) on perceived ambiguity and control variables. In sub-panels i and ii, the main independent variable $\log(\delta)$ is estimated using the data- and analyst-based methods, respectively. The regression is run for portfolio sorts in January 1996, 1995, and 1994. Columns (1)-(4) show the results with incremental addition of control variables, namely portfolio Sharpe ratio (Sharpe), skewness (Skew), and portfolio value in USD (log(PrtfValue)). ***, **, and *denote statistical significance at the 0.01, 0.05, and 0.10 levels, respectively, with p-values in parentheses.

(a) Div_1

	January 1996				January 1995				January 1994			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
(i) Data-based												
$\log(\delta)$	0.59***	0.52***	0.51***	0.46***	0.63***	0.58***	0.57***	0.52***	0.58***	0.52***	0.52***	0.47***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Sharpe		0.77***	0.74***	0.58***		0.48***	0.45***	0.32***		0.57***	0.55***	0.41***
		(0.00)	(0.00)	(0.00)		(0.00)	(0.00)	(0.00)		(0.00)	(0.00)	(0.00)
Skew			-0.03***	-0.02***			-0.02***	-0.01***			-0.01***	-0.01***
			(0.00)	(0.00)			(0.00)	(0.00)			(0.00)	(0.00)
log(PrtfValue)				0.03***				0.04***				0.04***
				(0.00)				(0.00)				(0.00)
Constant	-0.54***	-0.59***	-0.57***	-0.80***	-0.62***	-0.64***	-0.62***	-0.84***	-0.54***	-0.56***	-0.55***	-0.79***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Num. Obs.	18059	18059	18059	18059	22321	22321	22321	22321	28539	28539	28539	28539
Adj. R^2	0.46	0.51	0.52	0.54	0.48	0.50	0.50	0.52	0.46	0.50	0.50	0.52
(ii) Analyst-based												
$\log(\delta)$	0.58***	0.55***	0.54***	0.53***	0.56***	0.53***	0.53***	0.51***	0.55***	0.53***	0.52***	0.51***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Sharpe		0.61***	0.61***	0.57***		0.69***	0.67***	0.63***		0.63***	0.61***	0.58***
		(0.00)	(0.00)	(0.00)		(0.00)	(0.00)	(0.00)		(0.00)	(0.00)	(0.00)
Skew			-0.02***	-0.02***			-0.03***	-0.03***			-0.04***	-0.04***
			(0.00)	(0.00)			(0.00)	(0.00)			(0.00)	(0.00)
log(PrtValue)				0.01***				0.01***				0.01***
				(0.00)				(0.00)				(0.00)
Constant	-0.49***	-0.56***	-0.56***	-0.64***	-0.47***	-0.55***	-0.55***	-0.65***	-0.49***	-0.58***	-0.57***	-0.67***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Num. Obs.	8397	8397	8397	8397	10672	10672	10672	10672	13476	13476	13476	13476
Adj. R^2	0.73	0.77	0.77	0.77	0.72	0.76	0.76	0.77	0.70	0.75	0.76	0.76

Table 5: (continued)

(b) Div_2

	January 1996				January 1995				January 1994			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
						(i) Data-based						
log(δ)	0.33***	0.29***	0.28***	0.26***	0.34***	0.31***	0.31***	0.28***	0.31***	0.28***	0.28***	0.25***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Sharpe		0.39***	0.37***	0.29***		0.20***	0.19***	0.13***		0.26***	0.25***	0.18***
		(0.00)	(0.00)	(0.00)		(0.00)	(0.00)	(0.00)		(0.00)	(0.00)	(0.00)
Skew			-0.01***	-0.01***			-0.01***	-0.00**			-0.00***	-0.00
			(0.00)	(0.00)			(0.00)	(0.04)			(0.00)	(0.20)
log(PrtfValue)				0.02***				0.02***				0.02***
				(0.00)				(0.00)				(0.00)
Constant	-0.33***	-0.36***	-0.35***	-0.47***	-0.36***	-0.37***	-0.36***	-0.48***	-0.32***	-0.33***	-0.32***	-0.45***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Num. Obs.	18059	18059	18059	18059	22321	22321	22321	22321	28539	28539	28539	28539
Adj. R^2	0.43	0.48	0.48	0.50	0.45	0.46	0.46	0.48	0.43	0.45	0.45	0.48
						(ii) Analyst-based						
log(δ)	0.25***	0.24***	0.23***	0.23***	0.23***	0.22***	0.22***	0.21***	0.23***	0.22***	0.21***	0.21***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Sharpe		0.27***	0.27***	0.25***		0.28***	0.27***	0.24***		0.25***	0.23***	0.21***
		(0.00)	(0.00)	(0.00)		(0.00)	(0.00)	(0.00)		(0.00)	(0.00)	(0.00)
Skew			-0.01***	-0.01***			-0.02***	-0.02***			-0.02***	-0.02***
			(0.00)	(0.00)			(0.00)	(0.00)			(0.00)	(0.00)
log(PrtfValue)				0.01***				0.01***				0.01***
				(0.00)				(0.00)				(0.00)
Constant	-0.22***	-0.25***	-0.25***	-0.30***	-0.21***	-0.24***	-0.24***	-0.29***	-0.21***	-0.25***	-0.24***	-0.29***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Num. Obs.	8397	8397	8397	8397	10672	10672	10672	10672	13476	13476	13476	13476
Adj. R^2	0.64	0.67	0.68	0.68	0.63	0.66	0.67	0.67	0.62	0.66	0.66	0.67

Table 6: Ambiguity and equity home bias puzzle in emerging markets

This table reports the cross-over foreign ambiguity parameter δ_c and the ambiguity aversion parameter λ of the home investor for which the optimal allocations with the worst-case and continuous ambiguity aversion models, respectively, have home bias equal to the observed one. Also reported are the data- ($\delta_{d,f}$) and EPU-based ($\delta_{E,f}$) ambiguity estimates for the set of foreign markets for each country, the corresponding home estimates ($\delta_{d,h}, \delta_{E,h}$), and the relative ambiguity (m). Panel A and B report results with the data- and EPU-based ambiguity estimates, respectively. $\delta_c(m)$ and $\delta_c(\bar{m})$ are the cross-over values for relative ambiguity for each country (m) or the country average ($\bar{m}$). In panel A, the model selects portfolios from the sample of 21 developed and 19 emerging markets. The model is solved in panel B for the sub-sample of 15 developed and eight emerging markets of countries with EPU ratings. Countries without EPU data have missing entries marked -. The sample period spans January 1999–December 2019.

Country	$\delta_c(m)$	$\delta_c(\bar{m})$	$\delta_{d,f}$	$\delta_{d,h}$	m	λ	$\delta_c(m)$	$\delta_c(\bar{m})$	$\delta_{E,f}$	$\delta_{E,h}$	m	λ
	(a) Data-based						(b) EPU-based					
Brazil	1.4	1.4	12.49	4.61	0.37	0.58	1.5	1.5	11.13	1.07	0.10	0.58
Chile	2.5	2.5	12.24	4.53	0.37	0.60	2.0	2.0	8.19	0.81	0.10	0.63
China	2.5	2.5	12.38	4.28	0.35	0.58	2.0	2.0	8.91	1.45	0.16	0.63
Colombia	-	-	12.21	6.46	0.53	0.50	1.3	1.3	8.35	0.74	0.09	0.58
Czechia	2.5	2.4	12.41	5.08	0.41	0.56	-	-	-	-	-	-
Egypt	1.9	1.7	11.87	5.42	0.46	0.55	-	-	-	-	-	-
Hungary	2.8	3.4	12.16	3.42	0.28	0.58	-	-	-	-	-	-
India	2.6	2.6	12.20	3.98	0.33	0.55	2.0	2.0	8.87	0.72	0.08	0.63
Israel	3.8	3.2	11.94	2.94	0.25	0.63	-	-	-	-	-	-
Korea	3.1	3.4	12.07	2.68	0.22	0.57	2.3	2.2	8.04	0.94	0.12	0.65
Malaysia	2.8	3.1	12.65	2.48	0.20	0.60	-	-	-	-	-	-
Mexico	3.1	3	12.36	4.57	0.37	0.61	2.0	2.0	8.06	0.63	0.08	0.63
Peru	1.8	1.7	12.19	4.68	0.38	0.53	-	-	-	-	-	-
Philippines	3	2.9	11.23	4.19	0.37	0.61	-	-	-	-	-	-
Poland	4.5	5.1	12.23	3.96	0.32	0.63	-	-	-	-	-	-
Russia	-	2.1	11.37	6.10	0.54	0.58	3.0	0.7	8.61	1.05	0.12	0.55
South Africa	2.2	2.3	12.23	3.49	0.29	0.56	-	-	-	-	-	-
Thailand	2.6	2.4	12.12	2.67	0.22	0.53	-	-	-	-	-	-
Turkey	2.5	3.2	12.83	3.82	0.30	0.58	-	-	-	-	-	-
Mean	2.7	2.7	12.17	4.18	0.34	0.58	2.0	1.7	8.77	0.93	0.11	0.61
StdDev	0.7	0.8	0.38	1.11	0.10	0.03	0.5	0.5	1.01	0.27	0.03	0.04

Online Appendix

A Data Appendix

Table A.1: Descriptive statistics of equity home bias

This table reports the average home allocation (Home), market capitalization (Market), home allocation according to minimum variance without short-sales (MinV), and home bias indices with respect to market capitalization HBI(ICAPM) and minimum variance HBI(MinV). Data from the IMF CPIS database.

	Home	Market	MinV	HBI (ICAPM)	HBI (MinV)
(a) Developed markets					
Australia	0.80	0.021	0.19	0.80	0.76
Austria	0.59	0.002	0.00	0.59	0.59
Belgium	0.67	0.006	0.00	0.67	0.67
Canada	0.70	0.033	0.21	0.69	0.62
Denmark	0.58	0.004	0.00	0.58	0.58
Finland	0.67	0.004	0.00	0.67	0.67
France	0.76	0.039	0.00	0.75	0.76
Germany	0.72	0.030	0.00	0.71	0.72
Greece	0.96	0.002	0.00	0.96	0.96
Hong Kong	0.91	0.041	0.00	0.90	0.91
Italy	0.75	0.014	0.00	0.75	0.75
Japan	0.87	0.081	0.43	0.86	0.77
Netherlands	0.40	0.014	0.00	0.39	0.40
New Zealand	0.63	0.001	0.43	0.63	0.35
Norway	0.38	0.004	0.00	0.38	0.38
Portugal	0.77	0.002	0.07	0.77	0.76
Spain	0.90	0.020	0.00	0.90	0.90
Sweden	0.65	0.010	0.00	0.64	0.65
Switzerland	0.75	0.023	0.68	0.74	0.21
USA	0.84	0.396	0.23	0.73	0.78
UK	0.67	0.064	0.35	0.65	0.50
Mean	0.71	0.04	0.12	0.70	0.65
StdDev	0.15	0.08	0.19	0.14	0.19
(b) Emerging markets					
Brazil	0.99	0.015	0.46	0.99	0.98
Chile	0.87	0.004	0.36	0.87	0.80
China	0.99	0.081	0.00	0.99	0.99
Colombia	0.94	0.002	0.22	0.94	0.92
Czechia	0.89	0.001	0.14	0.89	0.87
Egypt	1.00	0.001	0.30	1.00	0.99
Hungary	0.85	0.001	0.09	0.85	0.83
India	1.00	0.022	0.01	1.00	1.00
Israel	0.84	0.003	0.11	0.84	0.82
Korea	0.93	0.017	0.00	0.93	0.93
Malaysia	0.96	0.006	0.28	0.96	0.95
Mexico	0.99	0.006	0.13	0.99	0.99
Peru	0.97	0.001	0.01	0.97	0.97
Philippines	1.00	0.002	0.16	1.00	1.00
Poland	0.98	0.002	0.06	0.98	0.98
Russia	1.00	0.012	0.03	1.00	1.00
South Africa	0.90	0.013	0.40	0.90	0.84
Thailand	0.99	0.005	0.00	0.99	0.99
Turkey	1.00	0.003	0.14	1.00	1.00
Mean	0.95	0.01	0.15	0.95	0.94
StdDev	0.06	0.02	0.15	0.06	0.07

Table A.2: Descriptive statistics of returns

This table reports descriptive statistics for the excess returns for all countries in our sample: mean, standard deviation, skewness, excess kurtosis, Value-at-Risk (VaR), Conditional-Value-at-Risk (CVaR), mean-to-CVAR (MtC) and Sharpe ratio for each country's monthly excess USD returns over the one-month USA T-Bill rate. It also displays the data- and EPU-based estimates of mean returns ambiguity for home $\delta_{d,h}$ and $\delta_{E,h}$, and foreign $\delta_{d,f}$ and $\delta_{E,f}$. Countries without EPU ratings have missing entries marked -. VaR, CVaR, and MtC are computed at the 95% confidence level. Mean, StdDev, VaR, and CVaR are in percentage points. The sample period spans January 1999–December 2019.

Country	Mean	StdDev	Skew	Kurt	VaR	CVaR	MtC	Sharpe	$\delta_{d,h}$	$\delta_{d,f}$	$\delta_{E,h}$	$\delta_{E,f}$
(a) Developed markets												
Australia	0.77	5.98	-0.54	1.99	8.35	13.77	0.06	0.13	3.92	12.21	0.71	7.07
Austria	0.60	6.81	-0.87	4.32	9.45	15.91	0.04	0.09	4.43	12.46	-	-
Belgium	0.35	6.00	-1.22	5.60	9.46	15.09	0.02	0.06	3.84	12.47	0.78	7.99
Canada	0.71	5.61	-0.53	2.62	8.39	12.09	0.06	0.13	3.71	12.34	1.14	7.66
Denmark	0.87	5.70	-0.73	2.69	9.38	13.63	0.06	0.15	2.96	12.48	0.80	8.75
Finland	0.60	8.11	0.10	2.07	13.42	18.13	0.03	0.07	2.87	12.47	-	-
France	0.49	5.80	-0.46	0.99	10.58	13.62	0.04	0.08	3.13	12.23	1.30	6.32
Germany	0.46	6.50	-0.37	1.64	10.25	15.48	0.03	0.07	3.13	12.24	1.02	7.06
Greece	-0.37	10.55	-0.23	0.68	18.01	24.24	-0.02	-0.03	4.17	12.37	0.88	8.56
Hong Kong	0.70	6.04	-0.17	1.46	9.77	13.12	0.05	0.12	3.23	12.31	0.94	9.16
Italy	0.24	6.61	-0.22	0.58	11.20	14.70	0.02	0.04	4.07	12.45	0.80	7.16
Japan	0.32	4.77	-0.12	0.33	7.98	9.91	0.03	0.07	4.29	12.20	0.80	10.67
Netherlands	0.46	5.76	-0.71	1.94	9.65	14.05	0.03	0.08	3.15	12.41	0.68	7.46
New Zealand	0.93	5.74	-0.44	0.79	8.72	12.55	0.07	0.16	4.26	12.10	-	-
Norway	0.86	7.28	-0.65	2.79	9.39	16.38	0.05	0.12	4.35	12.38	-	-
Portugal	0.09	6.30	-0.33	0.82	10.03	13.97	0.01	0.01	4.30	12.44	-	-
Spain	0.40	6.70	-0.14	1.04	10.08	14.31	0.03	0.06	3.71	12.46	0.85	7.74
Sweden	0.78	6.98	-0.15	1.93	11.70	16.00	0.05	0.11	3.41	12.33	0.67	7.75
Switzerland	0.51	4.43	-0.46	0.62	7.37	10.35	0.05	0.12	3.55	12.31	-	-
UK	0.33	4.67	-0.38	1.45	7.22	10.17	0.03	0.07	3.15	12.46	0.88	9.07
USA	0.52	4.33	-0.64	1.02	7.85	9.84	0.05	0.12	3.91	12.00	0.90	8.96
Mean	0.51	6.22	-0.44	1.78	9.92	14.16	0.04	0.09	3.69	12.34	0.88	8.09
StdDev	0.30	1.37	0.30	1.30	2.37	3.22	0.02	0.05	0.51	0.13	0.17	1.11
(b) Emerging markets												
Brazil	1.38	10.55	-0.04	1.16	14.06	21.93	0.06	0.13	4.61	12.49	1.07	11.13
Chile	0.67	6.26	-0.23	1.34	9.15	13.24	0.05	0.11	4.53	12.24	0.81	8.19
China	0.85	8.21	0.41	3.98	13.07	17.24	0.05	0.10	4.28	12.38	1.45	8.91
Colombia	1.15	8.20	-0.16	0.26	12.88	16.34	0.07	0.14	6.46	12.21	0.74	8.35
Czechia	1.02	7.43	-0.09	1.24	10.59	15.39	0.07	0.14	5.08	12.41	-	-
Egypt	0.79	8.93	0.07	2.14	13.41	18.50	0.04	0.09	5.42	11.87	-	-
Hungary	0.88	9.16	-0.51	2.19	14.60	21.38	0.04	0.10	3.42	12.16	-	-
India	1.12	8.28	-0.02	2.04	13.22	17.38	0.06	0.13	3.98	12.20	0.72	8.87
Israel	0.52	6.53	-0.18	1.32	11.96	14.77	0.04	0.08	2.94	11.94	-	-
Korea	0.95	8.50	0.20	0.92	13.94	16.61	0.06	0.11	2.68	12.07	0.94	8.04
Malaysia	0.75	5.78	0.63	4.58	9.01	11.37	0.07	0.13	2.48	12.65	-	-
Mexico	0.80	6.67	-0.50	1.58	10.62	14.55	0.05	0.12	4.57	12.36	0.63	8.06
Peru	1.19	7.64	-0.28	2.14	11.51	15.72	0.08	0.16	4.68	12.19	-	-
Philippines	0.57	6.95	-0.02	0.97	11.08	14.56	0.04	0.08	4.19	11.23	-	-
Poland	0.74	9.11	-0.10	0.79	13.16	18.98	0.04	0.08	3.96	12.23	-	-
Russia	1.91	10.59	0.55	3.44	15.09	20.26	0.09	0.18	6.10	11.37	1.05	8.61
South Africa	0.91	7.14	-0.31	0.10	10.62	14.36	0.06	0.13	3.49	12.23	-	-
Thailand	1.07	8.47	-0.01	2.92	11.46	18.95	0.06	0.13	2.67	12.12	-	-
Turkey	1.18	13.51	0.53	3.12	17.10	27.07	0.04	0.09	3.82	12.83	-	-
Mean	0.97	8.31	0.00	1.91	12.45	17.29	0.06	0.12	4.18	12.17	0.93	8.77
StdDev	0.32	1.83	0.33	1.23	2.07	3.64	0.01	0.03	1.11	0.38	0.27	1.01

Table A.3: Country data sources for the risk-free rates

This table reports the risk-free source for all countries in our sample. Data are from Datastream except for eurozone and USA, which are from Refinitiv and Kenneth French's website.

Country	Description
Australia	One-month Australian Dollar deposit rate
Brazil	Brazil interbank deposit certificates rate
Canada	One-month Canada Treasury Bill rate
Chile	90-days Chile Discountable Promissory Notes rate
China	One-month China Repo rate
Colombia	90-days Colombia certificate of deposit rate
Czechia	90-days Czech inter-bank delayed rate
Denmark	One-month Denmark inter-bank delayed rate
Egypt	One-month Egypt inter-bank rate
Euro area	One-month Euribor rate
Hong Kong	One-month Hong Kong inter-bank rate
Hungary	One-month Hungary inter-bank rate
India	Overnight India deposit rate
Israel	One-month Tel Aviv inter-bank rate
Japan	30-days Japan domestic banks deposit rate
Korea	One-month South Korea inter-bank rate
Malaysia	One-month Malaysia inter-bank rate
Mexico	28-days Mexico Cetes closing rate
New Zealand	One-month New Zealand Dollar deposit rate
Norway	One-month Norway inter-bank delayed rate
Peru	Peru inter-bank rate
Philippine	30-60 days Philippine time deposit rate
Poland	One-month Polish Zloty deposit rate
Russia	30-days Russia inter-bank actual credit rate
South Africa	One-month South African JIBAR rate
Sweden	30-days Sweden Treasury Bill rate
Switzerland	One-month Swiss Franc deposit rate
Thailand	One-month Thailand inter-bank (Bangkok Bank) rate
Turkey	One-month Turkey deposit rate
UK	One-month UK Treasury Bill Tender rate
USA	One-month USA Treasury Bill rate

Table A.4: Descriptive statistics of our subsamples of household portfolios

This table reports descriptive statistics of the number of portfolios of different sizes (number of stock holdings) in the original database (N) and the two sub-samples constructed to obtain data- (N_d) and analyst-based (N_a) ambiguity estimates for January 1996, 1995, and 1994.

Portfolio size	N	N_d	N_a	N	N_d	N_a	N	N_d	N_a
	(a) January 1996			(b) January 1995			(c) January 1994		
1	6254	4328	1951	7567	5494	2482	9680	7377	3253
2	3997	3376	1631	4900	4254	2104	6133	5423	2710
3	2759	2411	1194	3330	3027	1590	4416	4091	2054
4	2099	1840	925	2573	2352	1213	3112	2893	1444
5	1509	1315	657	1779	1624	820	2350	2132	1078
6-9	3404	2816	1338	3966	3430	1625	4692	4141	1976
10+	3074	1973	701	3133	2140	838	3677	2482	961
All	23096	18059	8397	27248	22321	10672	34060	28539	13476

Table A.5: Descriptive statistics of household portfolios

This table reports descriptive statistics of the diversification, ambiguity, and returns of our samples of household portfolios. Panels A and B present the statistics for the data- and analyst-based samples. We report statistics of the diversification (Div_1 and Div_2), the estimated portfolio ambiguity (δ), the expected portfolio return (Mean), standard deviation (StdDev), and Skewness (Skew). Statistics are reported for January 1996, 1995, and 1994 and are computed using monthly returns over the preceding ten years. The data- and analyst-based sample for three dates include 18,059, 22,321, 28,539, and 8,397, 10,672, and 13,476 households, respectively.

	January 1996						January 1995						January 1994					
	Div_1	Div_2	δ	Mean	StdDev	Skew	Div_1	Div_2	δ	Mean	StdDev	Skew	Div_1	Div_2	δ	Mean	StdDev	Skew
(a) Data-based																		
Mean	0.37	0.17	4.95	0.018	0.088	0.090	0.37	0.16	5.05	0.016	0.088	0.112	0.37	0.17	5.09	0.016	0.089	0.074
StdDev	0.32	0.18	1.73	0.010	0.049	0.686	0.32	0.18	1.70	0.010	0.051	0.746	0.32	0.18	1.83	0.010	0.050	0.764
Min	0.00	0.00	1.51	-0.042	0.025	-2.040	0.00	0.00	1.74	-0.064	0.026	-2.900	0.00	0.00	1.58	-0.064	0.025	-1.910
25th	0.00	0.00	3.68	0.012	0.057	-0.301	0.00	0.00	3.78	0.011	0.057	-0.293	0.00	0.00	3.78	0.012	0.057	-0.359
Median	0.42	0.12	4.75	0.016	0.074	-0.041	0.43	0.12	4.81	0.015	0.073	-0.035	0.43	0.12	4.88	0.016	0.074	-0.067
75th	0.66	0.31	6.09	0.022	0.106	0.329	0.66	0.30	6.21	0.020	0.105	0.322	0.66	0.30	6.28	0.020	0.106	0.296
Max	0.96	0.79	9.99	0.104	0.946	9.910	0.96	0.76	9.96	0.180	1.830	9.760	0.96	0.73	10.60	0.136	0.860	8.940
(b) Analyst-based																		
Mean	0.24	0.09	3.93	0.013	0.076	-0.082	0.24	0.09	3.99	0.012	0.076	-0.073	0.24	0.09	4.12	0.013	0.075	-0.134
StdDev	0.29	0.13	1.94	0.006	0.031	0.473	0.29	0.13	2.02	0.006	0.031	0.481	0.29	0.13	2.05	0.006	0.030	0.437
Min	0.00	0.00	1.97	-0.015	0.029	-2.360	0.00	0.00	1.90	-0.015	0.032	-1.970	0.00	0.00	2.01	-0.019	0.030	-2.030
25th	0.00	0.00	2.51	0.009	0.051	-0.326	0.00	0.00	2.42	0.008	0.052	-0.353	0.00	0.00	2.53	0.010	0.053	-0.410
Median	0.00	0.00	3.22	0.013	0.071	-0.135	0.00	0.00	3.35	0.012	0.071	-0.126	0.00	0.00	3.59	0.014	0.069	-0.198
75th	0.49	0.16	4.82	0.016	0.090	0.171	0.50	0.16	4.92	0.015	0.089	0.152	0.49	0.16	5.08	0.016	0.086	0.107
Max	0.93	0.64	11.40	0.058	0.218	4.120	0.92	0.63	11.80	0.057	0.229	3.700	0.91	0.59	12.00	0.035	0.198	2.070

B Supplementary Figures and Tables

Figure B.1: Equity home bias of broad market categories over time

This figure displays the equity home bias index (eqn. 14) against the market capitalization weights over time for the sample of developed, emerging, and world markets. The data are aggregated for each point in time by taking the average value across countries in the corresponding market category. The sample covers 21 developed and 19 emerging markets using annual data from the IMF CPIS database.

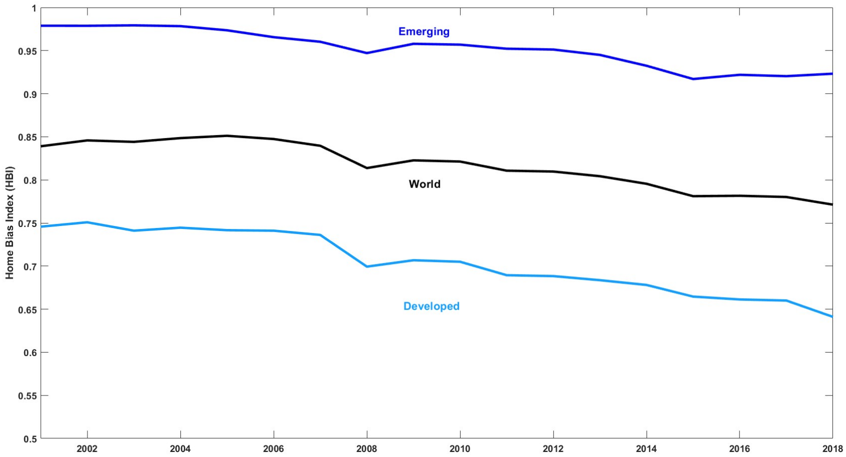
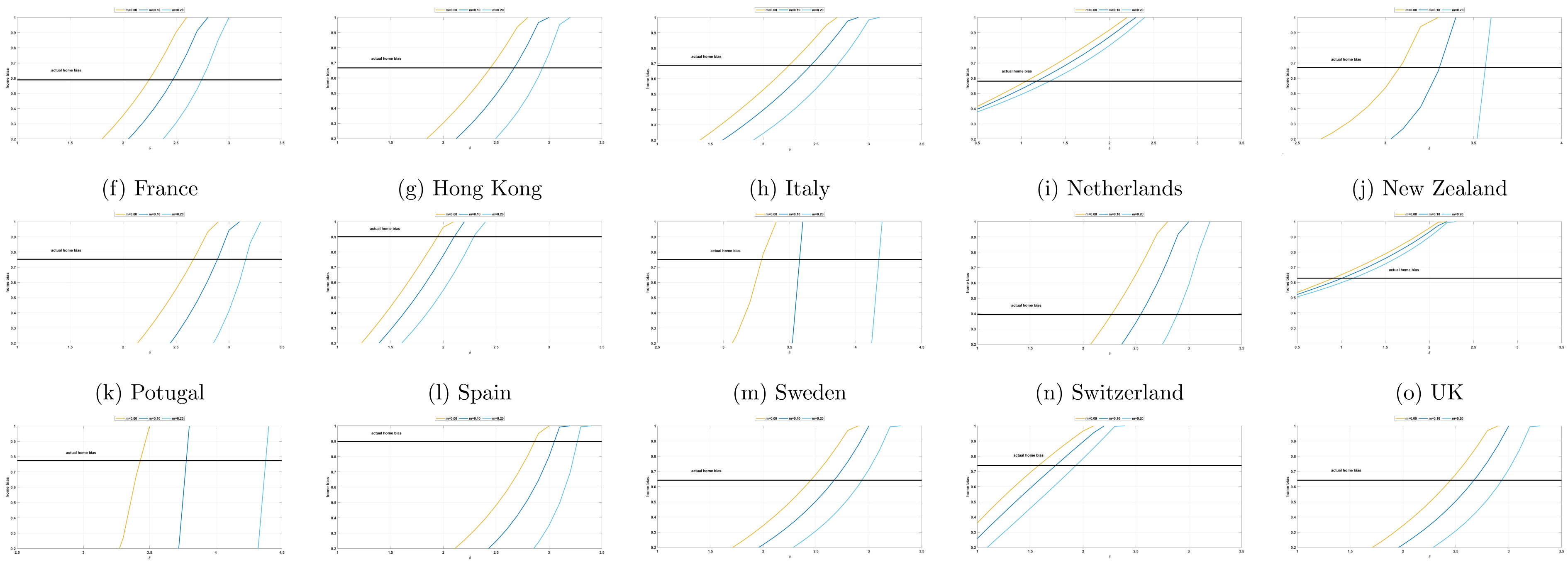


Figure B.2: Optimal home allocation and ambiguity for additional countries

This figure illustrates the home bias index (HBI) of the worst-case model optimal allocations with respect to the market capitalization as a function of foreign ambiguity δ_f for different relative ambiguity of home to foreign (m). The horizontal line indicates the observed time-average home bias estimated using the actual home allocation weights. The results are for investors in developed markets, and the model selects portfolios from the sample of 21 developed and 19 emerging markets. The sample period spans January 1999–December 2019. We do not display Greece, for which the model only explains the equity home bias with the EPU-based ambiguity estimates. With the data-based estimates, the crossover happens at a high $\delta > 15$ if the relative ambiguity is set to $m = 0.30$, and we need to decrease the relative ambiguity to 0.15 to obtain a crossover within the data-based ambiguity estimate of $\delta = 10.4$.



(a) Austria (b) Belgium (c) Canada (d) Denmark (e) Finland (f) France (g) Hong Kong (h) Italy (i) Netherlands (j) New Zealand (k) Potugal (l) Spain (m) Sweden (n) Switzerland (o) UK

C Background results and proofs

C.1 Background

We provide some general background material on CVaR and necessary definitions.

Definition C.1 (Conditional Value-at-Risk)**.** *The conditional Value-at-Risk at confidence level* $\alpha \in (0, 1)$, *for the random variable portfolio return* $\tilde{r}_p$ *is*

$$\mathrm{CVaR}_\alpha(\tilde{r}_p) = -\mathbb{E}[\tilde{r}_p \mid \tilde{r}_p \leq \zeta], \tag{C.1}$$

where $\mathbb{E}$ *is the expectation operator and* $\zeta \in \mathbb{R}$ *is the Value-at-Risk, i.e., the* $(1 - \alpha)$*-quantile of* $\tilde{r}_p$ *given by the highest* γ *such that* $\tilde{r}_p$ *will not exceed* γ *with probability* $1 - \alpha$,

$$\mathrm{VaR}_\alpha(\tilde{r}_p) \doteq \zeta = \max\{\gamma \in \mathbb{R} \mid \mathrm{Prob}(\tilde{r}_p \leq \gamma) \leq 1 - \alpha\}. \tag{C.2}$$

Theorem C.1 (Fundamental minimization formula (Rockafellar and Uryasev, 2002))**.** *As a function of* $\gamma \in \mathbb{R}$, *the auxiliary function*

$$F_\alpha(\tilde{r}_p, \gamma) = \gamma + \frac{1}{1-\alpha}\mathbb{E}\big[\max\{-\tilde{r}_p - \gamma, 0\}\big]$$

is finite and convex, with

$$\mathrm{CVaR}_\alpha(\tilde{r}_p) = \min_{\gamma \in \mathbb{R}} F_\alpha(\tilde{r}_p, \gamma).$$

For convenience, we drop the parameter α from our use of CVaR, which we set at 0.95 in all numerical tests. We note that Rockafellar and Uryasev (2002) develop their model for a random loss variable $\tilde{z}$ and not for returns. Their CVaR of losses is the expected value *above* a threshold ζ. In contrast, we take the CVaR of return as the negative of the expected value of returns *below* the $1 - \alpha$ probability threshold ζ. We use their results with $\tilde{z} = -\tilde{r}_p$ to develop our model in returns.

Definition C.2 (Stochastic dominance (Ogryczak and Ruszczyński, 2002))**.** *Random variable* $\tilde{X}$ *dominates random variable* $\tilde{Y}$ *under first order stochastic dominance (FSD,* $\tilde{X} \succeq_{FSD} \tilde{Y}$*) if* $\mathbb{E}(U(\tilde{X})) \geq \mathbb{E}(U(\tilde{Y}))$ *for all non-decreasing utility functions* U*. Similarly,* $\tilde{X}$ *dominates random variable* $\tilde{Y}$ *under second order stochastic dominance (SSD,* $\tilde{X} \succeq_{SSD} \tilde{Y}$*) if* $\mathbb{E}(U(\tilde{X})) \geq \mathbb{E}(U(\tilde{Y}))$ *for all non-decreasing concave utility functions* U*.*

Definition C.3 (Risk measure consistency (Ogryczak and Ruszczyński, 2002))**.** *Given a stochastic order* $\succeq_{SSD}$ *we say that a risk measure* ρ *is SSD consistent if* $\tilde{X} \succeq_{SSD} \tilde{Y}$ *implies* $\rho(\tilde{X}) \leq \rho(\tilde{Y})$*.*

Definition C.4 (Worst case risk measure (Zhu and Fukushima, 2009)). *Assume the random variable $\tilde{X}$ with π indicating its probability distribution and the corresponding risk measure $\rho(\tilde{X})$ are given. Further the probability measure π is ambiguous and characterized with an ambiguity set $\mathcal{P}$, then we define the worst case risk measure as follows:*

$$\rho_w(\tilde{X}) = \underset{\pi \in \mathcal{P}}{sup}\ \rho(\tilde{X}).$$

C.2 Proof of Theorem 2.1

First, we establish a proposition on the SSD consistency of worst-case risk measures needed for our proof.

Proposition C.1 (SSD consistency of worst-case risk measures). *If the risk measure associated with probability distribution π is SSD consistent, then the worst-case risk measure ρ_w associated with distribution ambiguity set $\mathcal{P}$ remains SSD consistent.*

Proof

Assume random variables $\tilde{X}$ and $\tilde{Y}$ are arbitrary given and $\tilde{X}$ dominates $\tilde{Y}$, or equivalently, $\tilde{X} \succeq_{SSD} \tilde{Y}$. That is, $\tilde{X}$ is preferred to $\tilde{Y}$ within all risk-averse preference models with a non-decreasing and concave utility function. Since the risk measure ρ is SSD consistent then $\rho(\tilde{X}) \leq \rho(\tilde{Y})$, and therefore $\rho_w(\tilde{X}) = \max_{\pi \in \mathcal{P}} \rho(\tilde{X}) \leq \max_{\pi \in \mathcal{P}} \rho(\tilde{Y}) = \rho_w(\tilde{Y})$. That completes the proof of the proposition. □

Now we prove the Theorem. Let us assume the portfolios w_1 and w_0 belong to $\mathbb{X}_+$, and w_1 dominates w_0 i.e., the portfolios' returns satisfy $\tilde{r}_{w_1} \succeq_{SSD} \tilde{r}_{w_0}$ with $\tilde{r}_w = \tilde{r}^\top w$, for any arbitrary probability distribution of $\tilde{r}$ in $\mathbb{D}$ with mean returns, $\bar{r}_h$ and $\bar{r}_f$, in U_h and U_f, respectively. This implies $\bar{r}^\top w_1 \geq \bar{r}^\top w_0 > 0$ (Whang, 2019, Theorem 1.1.5), or, equivalently, $\bar{r}^\top w_1 - r_f \geq \bar{r}^\top w_0 - r_f > 0$. Therefore, the worst-case mean excess returns of portfolios w_1 and w_0 satisfy the following inequality.

$$\begin{aligned}\min_{\substack{\bar{r}_h \in U_h \\ \bar{r}_f \in U_f}} \min_{\pi \in \mathbb{D}} \mathbb{E}(\tilde{r}_{w_1} - r_f) \ &= \min_{\substack{\bar{r}_h \in U_h \\ \bar{r}_f \in U_f}} \bar{r}^\top w_1 - r_f \geq \\ &\min_{\substack{\bar{r}_h \in U_h \\ \bar{r}_f \in U_f}} \bar{r}^\top w_0 - r_f = \min_{\substack{\bar{r}_h \in U_h \\ \bar{r}_f \in U_f}} \min_{\pi \in \mathbb{D}} \mathbb{E}(\tilde{r}_{w_0} - r_f).\end{aligned}$$

CVaR is SSD consistent (Ogryczak and Ruszczyński, 2002, Theorem 3.2). Proposition C.1 implies that worst-case CVaR of excess return is SSD consistent, or,

$$\max_{\substack{\bar{r}_h \in U_h \\ \bar{r}_f \in U_f}} \max_{\pi \in \mathbb{D}} \mathrm{CVaR}(\tilde{r}_{w_1} - r_f) \leq \max_{\substack{\bar{r}_h \in U_h \\ \bar{r}_f \in U_f}} \max_{\pi \in \mathbb{D}} \mathrm{CVaR}(\tilde{r}_{w_0} - r_f).$$

Note that under given assumptions, both worst-case CVaR of excess return and mean excess return of portfolios w_1 and w_0 are positive. Therefore, the above inequalities, taken together, imply that the ratio of worst-case CVaR to worst-case mean excess return for portfolio w_1 is less than or equal to the ratio of worst-case CVaR to worst-case mean excess return of portfolio w_0. That means the worst-case CVaR-to-mean ratio of portfolio w_1 is less than or equal to the worst-case CVaR-to-mean ratio of w_0. Hence, the inverse of the worst-case MtC ratio is SSD consistent. Since worst-case MtC is positive, one can easily see that $\rho(\tilde{X}) \leq \rho(\tilde{Y})$ is equivalent to $\frac{1}{\rho(\tilde{Y})} \leq \frac{1}{\rho(\tilde{X})}$ in Definition of risk measure consistency (see Definition C.3). Therefore, the worst-case MtC is SSD consistent. $\square$

C.3 Proof of Theorem 2.3

The second-order cone program with two assets is obtained from Theorem 2.2:

$$
\begin{aligned}
\max_{w'_h, w'_f \in \mathbb{R}_+} \quad & ((\hat{r}_h - r_f) - \delta_h \hat{\sigma}_h) w'_h + ((\hat{r}_f - r_f) - \delta_f \hat{\sigma}_f) w'_f \\
\text{s.t.} \quad & -((\hat{r}_h - r_f) - \delta_h \hat{\sigma}_h) w'_h - ((\hat{r}_f - r_f) - \delta_f \hat{\sigma}_f) w'_f \\
& + \sqrt{\frac{\alpha}{1-\alpha}} \sqrt{w'^2_h \hat{\sigma}^2_h + 2 w'_h w'_f \hat{\sigma}_h \hat{\sigma}_f \rho + w'^2_f \hat{\sigma}^2_f} \leq 1 \\
& w'_h + w'_f > 0.
\end{aligned} \tag{C.3}
$$

Since optimization model (C.3) is convex and satisfies the Slater regularity condition, the KKT optimality conditions are the necessary and sufficient condition for optimality. Let us define $\hat{\sigma}_p^2 = w'^{\star 2}_h \hat{\sigma}_h^2 + w'^{\star 2}_f \hat{\sigma}_f^2 + 2 w'^{\star}_h w'^{\star}_h \rho \hat{\sigma}_h \hat{\sigma}_f$ where $w'^{\star}_h$ and $w'^{\star}_f$ represents the optimal solutions. The KKT optimality conditions are as follows:

$$
\begin{aligned}
(-\hat{r}_f + r_f + \delta_f \hat{\sigma}_f) + \phi \left((-\hat{r}_f + r_f + \delta_f \hat{\sigma}_f) + \frac{\sqrt{\alpha}}{\hat{\sigma}_p \sqrt{1-\alpha}} (\hat{\sigma}_f^2 w'^{\star}_f + \rho \hat{\sigma}_h \hat{\sigma}_f w'^{\star}_h) \right) &= 0 \\
(-\hat{r}_h + r_f + \delta_h \hat{\sigma}_h) + \phi \left((-\hat{r}_h + r_f + \delta_h \hat{\sigma}_h) + \frac{\sqrt{\alpha}}{\hat{\sigma}_p \sqrt{1-\alpha}} (\hat{\sigma}_h^2 w'^{\star}_h + \rho \hat{\sigma}_h \hat{\sigma}_f w'^{\star}_f) \right) &= 0 \\
\phi \left(1 + (\hat{r}_f - r_f - \delta_f \hat{\sigma}_f) w'^{\star}_f + (\hat{r}_h - r_f - \delta_h \hat{\sigma}_h) w'^{\star}_h - \sqrt{\frac{\alpha}{1-\alpha}} \hat{\sigma}_p \right) &= 0
\end{aligned} \tag{C.4}
$$

where $\phi \geq 0$ is the Lagrange multiplier.

By obtaining ϕ from the first two KKT conditions and equating them together, we get

$$
(\hat{r}_f - r_f - \delta_f \hat{\sigma}_f)(\hat{\sigma}_h^2 w'^{\star}_h + \rho \hat{\sigma}_h \hat{\sigma}_f w'^{\star}_f) - (\hat{r}_h - r_f - \delta_h \hat{\sigma}_h)(\hat{\sigma}_f^2 w'^{\star}_f + \rho \hat{\sigma}_h \hat{\sigma}_f w'^{\star}_h) = 0.
$$

Dividing above by $\hat{\sigma}_h \hat{\sigma}_f$, and using s to denote the Sharpe ratio we get

$$
(s_f - \delta_f) \hat{\sigma}_h w'^{\star}_h + (s_f - \delta_f) \rho \hat{\sigma}_f w'^{\star}_f - (s_h - \delta_h) \hat{\sigma}_f w'^{\star}_f - (s_h - \delta_h) \rho \hat{\sigma}_h w'^{\star}_h = 0,
$$

or,

$$((s_f - \delta_f) - \rho(s_h - \delta_h))\hat{\sigma}_h w'^{\star}_h - ((s_h - \delta_h) - \rho(s_f - \delta_f))\hat{\sigma}_f w'^{\star}_f = 0. \tag{C.5}$$

If both aa-premia are positive, and for $\rho > 0$, one of the following cases is possible:

i) $\rho(s_f - \delta_f) < (s_h - \delta_h) < \frac{1}{\rho}(s_f - \delta_f)$.

Then eqn. (C.5) implies both $w'^{\star}_h$ and $w'^{\star}_f$ are positive. Dividing it by $w'^{\star}_h + w'^{\star}_f > 0$,

$$((s_f - \delta_f) - \rho(s_h - \delta_h))\hat{\sigma}_h w^{\star}_h - ((s_h - \delta_h) - \rho(s_f - \delta_f))\hat{\sigma}_f w^{\star}_f = 0,$$

where $w^{\star}_h$ and $w^{\star}_f$ are the optimal solution of model (6). As $w^{\star}_f = 1 - w^{\star}_h$, we have

$$w^{\star}_h = \frac{((s_h - \delta_h) - \rho(s_f - \delta_f))\hat{\sigma}_f}{((s_f - \delta_f) - \rho(s_h - \delta_h))\hat{\sigma}_h + ((s_h - \delta_h) - \rho(s_f - \delta_f))\hat{\sigma}_f}.$$

ii) $(s_h - \delta_h) \leq \rho(s_f - \delta_f)$ or $(s_h - \delta_h) \geq \frac{1}{\rho}(s_f - \delta_f)$.

Then eqn. (C.5) implies the model (C.3) has a solution with one of $w'^{\star}_h$ and $w'^{\star}_f$ being positive. By deriving two candidate solutions from the first constrain of the model (C.3) and comparing the corresponding objective values, one can see that if $(s_h - \delta_h) \leq \rho(s_f - \delta_f)$, then $w'^{\star}_f > 0$ and $w'^{\star}_h = 0$ and if $(s_h - \delta_h) \geq \frac{1}{\rho}(s_f - \delta_f)$, then $w'^{\star}_h > 0$ and $w'^{\star}_f = 0$. This implies model (6) has a solution of $w^{\star}_h = 0$ if $(s_h - \delta_h) \leq \rho(s_f - \delta_f)$ and $w^{\star}_h = 1$ if $(s_h - \delta_h) \geq \frac{1}{\rho}(s_f - \delta_f)$.

If only one asset has a positive aa-premium, then case (ii) applies, and therefore the model (6) has a trivial solution of $w^{\star}_h = 0$ if $s_f - \delta_f > 0$ and $w^{\star}_h = 1$ if $s_h - \delta_h > 0$. □

C.4 Proof of Corollary 2.1

From Theorem 2.3, the optimal allocation is determined by the following two cases:

i) $\rho(s_f - \delta_f) < (s_h - \delta_h) < \frac{1}{\rho}(s_f - \delta_f)$.

The $w^{\star}_h$ is specified by eqn. (9). Taking derivatives from $w^{\star}_h$ with respect to δ_h we have

$$\frac{\partial w^{\star}_h}{\partial \delta_h} = \frac{1}{d^2}\left(-\hat{\sigma}_f d - (\rho\hat{\sigma}_h\hat{\sigma}_f - \hat{\sigma}_f^2)((s_h - \delta_h) - \rho(s_f - \delta_f))\right) \tag{C.6}$$

where d is the denominator of $w^{\star}_h$. By replacing d and canceling similar terms, we have

$$\frac{\partial w^{\star}_h}{\partial \delta_h} = \frac{\rho^2\hat{\sigma}_f\hat{\sigma}_h(s_f - \delta_f) - \hat{\sigma}_f\hat{\sigma}_h(s_f - \delta_f)}{\left[((s_f - \delta_f) - \rho(s_h - \delta_h))\hat{\sigma}_h + ((s_h - \delta_h) - \rho(s_f - \delta_f))\hat{\sigma}_f\right]^2},$$

or

$$\frac{\partial w^{\star}_h}{\partial \delta_h} = \frac{\hat{\sigma}_h\hat{\sigma}_f(1 - \rho^2)(\delta_f - s_f)}{\left[((s_f - \delta_f) - \rho(s_h - \delta_h))\hat{\sigma}_h + ((s_h - \delta_h) - \rho(s_f - \delta_f))\hat{\sigma}_f\right]^2}.$$

This is eqn. (10). By taking partial derivatives from $w_h^\star$ with respect to δ_f and following similar steps, one can obtain eqn. (11).

ii) $(s_h - \delta_h) \leq \rho(s_f - \delta_f)$ or $(s_h - \delta_h) \geq \frac{1}{\rho}(s_f - \delta_f)$.

Derivative of $w_h^\star$ with respect to δ_h and δ_f is zero as $w_h^\star$ is constant (Theorem 2.3, case ii). □

C.5 Ambiguity effects on diversifcation

We give the ambiguity effect on the diversification measure Div_1.

Corollary C.1. *For $\rho > 0$, the partial derivatives of* Div_1 *with respect to δ_h and δ_f at the optimal allocation of the model (6) are:*

i. If $\rho(s_f - \delta_f) < (s_h - \delta_h) < \frac{1}{\rho}(s_f - \delta_f)$

$$\frac{\partial \mathrm{Div}_1}{\partial \delta_h} = 2\hat{\sigma}_h\hat{\sigma}_f(1-\rho^2)(\delta_f - s_f)F \tag{C.7}$$

$$\frac{\partial \mathrm{Div}_1}{\partial \delta_f} = 2\hat{\sigma}_h\hat{\sigma}_f(1-\rho^2)(s_h - \delta_h)F, \tag{C.8}$$

with

$$F = \frac{((s_f-\delta_f)-\rho(s_h-\delta_h))\,\hat{\sigma}_h - ((s_h-\delta_h)-\rho(s_f-\delta_f))\,\hat{\sigma}_f}{\left[((s_f-\delta_f)-\rho(s_h-\delta_h))\,\hat{\sigma}_h + ((s_h-\delta_h)-\rho(s_f-\delta_f))\,\hat{\sigma}_f\right]^3}. \tag{C.9}$$

ii. Zero, if $(s_h - \delta_h) \leq \rho(s_f - \delta_f)$ or $(s_h - \delta_h) \geq \frac{1}{\rho}(s_f - \delta_f)$.

Proof

Given the definition of Div_1, derivative of Div_1 with respect to δ_h is as follows:

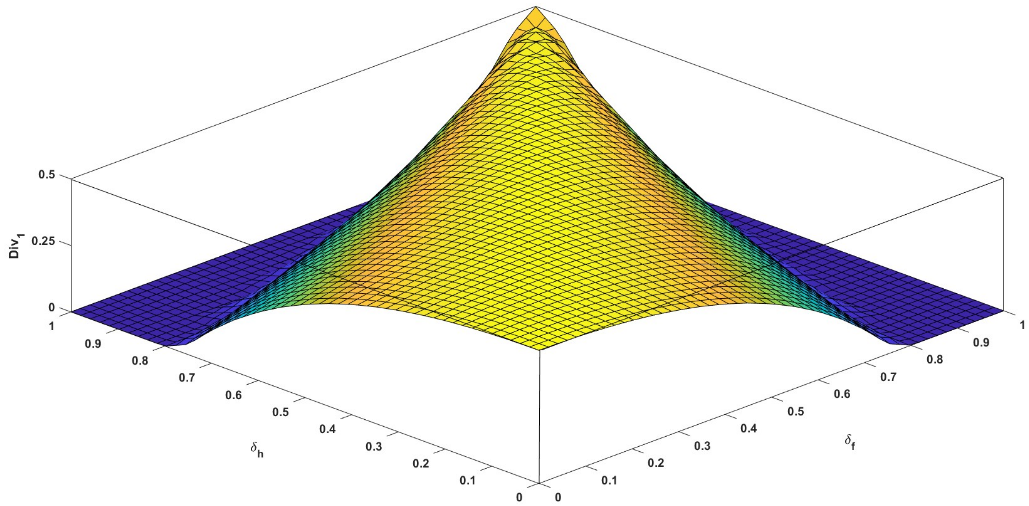
$$\frac{\partial \mathrm{Div}_1}{\partial \delta_h} = (1 - 2w_h^\star)\frac{\partial w_h^\star}{\partial \delta_h}. \tag{C.10}$$

Replacing the $w_h^\star$ from Theorem 2.3 and $\frac{\partial w_h^\star}{\partial \delta_h}$ from Corollary 2.1, we get the result. Similar procedure can be followed to obtain $\frac{\partial \mathrm{Div}_1}{\partial \delta_f}$. □

For $\rho \leq 0$ case (i) of Corollary C.1 applies. The sign of the partial derivative of Div_1 with respect to δ_f is positive for $F > 0$ and negative for $F < 0$. When the means and standard deviations are the same, then $\frac{\partial \mathrm{Div}_1}{\partial \delta_f} < 0$ for $\delta_f > \delta_h$, so that increasing ambiguity induces under-diversification, everything else being equal. For equal ambiguities, the partial derivative is zero, and we achieve maximum diversification. There is a nice symmetry around the maximum, illustrated in Figure C.1 that displays the model's diversification between two assets assumed to have identical expected returns (0.6%) and standard deviation (4.3%) with a correlation of 0.3. With increasing ambiguity, the allocation shifts towards one of the assets (diversification 0) depending on which market is more ambiguous. For equal ambiguities, the allocation is perfectly diversified (diversification 0.5). This surface twists with heterogeneous ambiguity sets among multiple assets and with asset returns correlation.

Figure C.1: Diversification among two correlated assets with heterogenous ambiguity

This figure illustrates the diversification of optimal asset allocation using the worst-case model (6) for two assets with identical excess monthly return means (0.6%) and standard deviations (4.3%), scaled ambiguity parameters δ_h and δ_f in the range 0 to 1, and correlation $\rho = 0.3$.



D Second-order cone program formulations

D.1 Worst-case ambiguity aversion

Consider the MtC model (2). We assume that the CVaR of portfolio excess return is positive for any portfolio in $\mathbb{X}$. This is a reasonable assumption, given that CVaR denotes losses and for α large enough any portfolio will have positive losses at the tail. Defining $\xi = \mathrm{CVaR}(\tilde{r}_p - r_f) > 0$, we write the MtC model as:

$$\begin{aligned} \max_{w\in\mathbb{X},\, \xi\in\mathbb{R}} \quad & \frac{1}{\xi}(\mathbb{E}(\tilde{r}) - r_f e)^\top w \\ \text{s.t.} \quad & \mathrm{CVaR}_\alpha((\tilde{r} - r_f e)^\top w) \leq \xi \\ & \xi > 0. \end{aligned} \tag{D.1}$$

Set $w' = \frac{w}{\xi}$. By positivity of ξ and positive homogeneity of CVaR, we write the above as:

$$
\begin{aligned}
\max_{w' \in \mathbb{R}^n_+} \quad & (\mathbb{E}(\tilde{r}) - r_f e)^\top w' \\
\text{s.t.} \quad & \text{CVaR}((\tilde{r} - r_f e)^\top w') \leq 1 \\
& e^\top w' > 0,
\end{aligned}
\tag{D.2}
$$

where $e^\top w' = \frac{1}{\xi}$, and thus $w = \frac{1}{e^\top w'} w'$. Introducing the heterogeneous ambiguity sets, and letting $w' = (w'_h, w'_f)$ be the concatenation of home and foreign allocations $w'_h \in \mathbb{R}_+$ and $w'_f \in \mathbb{R}^{n-1}_+$, we write the worst-case model as follows:

$$
\begin{aligned}
\max_{w' \in \mathbb{R}^n_+} \quad & \min_{\substack{\bar{r}_h \in U_h \\ \bar{r}_f \in U_f}} \min_{\pi \in \mathbb{D}} \ (\mathbb{E}(\tilde{r}) - r_f e)^\top w' \\
\text{s.t.} \quad & \max_{\substack{\bar{r}_h \in U_h \\ \bar{r}_f \in U_f}} \max_{\pi \in \mathbb{D}} \ \text{CVaR}((\tilde{r} - r_f e)^\top w') \leq 1 \\
& e^\top w' > 0.
\end{aligned}
\tag{D.3}
$$

Replacing CVaR from the fundamental minimization formula (Appendix C.1) we have:

$$
\begin{aligned}
\max_{w' \in \mathbb{R}^n_+} \quad & \min_{\substack{\bar{r}_h \in U_h \\ \bar{r}_f \in U_f}} \min_{\pi \in \mathbb{D}} \ (\mathbb{E}(\tilde{r}) - r_f e)^\top w' \\
\text{s.t.} \quad & \max_{\substack{\bar{r}_h \in U_h \\ \bar{r}_f \in U_f}} \max_{\pi \in \mathbb{D}} \min_{\gamma \in \mathbb{R}} \ F_\alpha((\tilde{r} - r_f e)^\top w', \gamma) \leq 1, \\
& e^\top w' > 0.
\end{aligned}
\tag{D.4}
$$

Obviously, $\min\limits_{\pi \in \mathbb{D}} (\mathbb{E}(\tilde{r}) - r_f e)^\top w' = (\bar{r} - r_f e)^\top w'$. Also, the max-min optimization in the first constraint can be obtained from Proposition 1 in Lotfi and Zenios (2018). Therefore, the above formulation can be written as follows:

$$
\begin{aligned}
\max_{w' \in \mathbb{R}^n_+} \quad & \min_{\substack{\bar{r}_h \in U_h \\ \bar{r}_f \in U_f}} (\bar{r} - r_f e)^\top w' \\
\text{s.t.} \quad & \max_{\substack{\bar{r}_h \in U_h \\ \bar{r}_f \in U_f}} \ -(\bar{r} - r_f e)^\top w' + \frac{\sqrt{\alpha}}{\sqrt{1-\alpha}} \sqrt{w'^\top \hat{\Sigma} w'} \leq 1 \\
& e^\top w' > 0.
\end{aligned}
\tag{D.5}
$$

In terms of home and foreign allocations we have:

$$
\begin{aligned}
\max_{(w'_h, w'_f)\in\mathbb{R}^n_+} \quad & \left(\min_{\bar{r}_h\in U_h} (\bar{r}_h - r_f)w'_h\right) + \left(\min_{\bar{r}_f\in U_f} \ (\bar{r}_f - r_f e)^\top w'_f\right) \\
\text{s.t.} \quad & -\left(\min_{\bar{r}_h\in U_h} (\bar{r}_h - r_f)w'_h\right) - \left(\min_{\bar{r}_f\in U_f} (\bar{r}_f - r_f e)^\top w'_f\right) + \\
& \frac{\sqrt{\alpha}}{\sqrt{1-\alpha}}\sqrt{w'^2_h\hat{\sigma}^2_h + 2w'_h\hat{\sigma}^\top_{hf}w'_f + w'^\top_f\hat{\Sigma}_f w'_f} \le 1 \\
& w'_h + e^\top w'_f > 0.
\end{aligned} \tag{D.6}
$$

One can check that $\min_{\bar{r}_f\in U_f} (\bar{r}_f - r_f e)^\top w'_f$ is equal to $(\hat{r}_f - r_f e)^\top w'_f - \delta_f\sqrt{w'^\top_f\hat{\Sigma}_f w'_f}$, and substituting above leads to the following formulation:

$$
\begin{aligned}
\max_{(w'_h, w'_f)\in\mathbb{R}^n_+} \quad & \left(\min_{\bar{r}_h\in U_h} \ (\bar{r}_h - r_f)w'_h\right) + (\hat{r}_f - r_f e)^\top w'_f - \delta_f\sqrt{w'^\top_f\hat{\Sigma}_f w'_f} \\
\text{s.t.} \quad & -\left(\min_{\bar{r}_h\in U_h} \ (\bar{r}_h - r_f)w'_h\right) - (\hat{r}_f - r_f e)^\top w'_f + \delta_f\sqrt{w'^\top_f\hat{\Sigma}_f w'_f} + \\
& \frac{\sqrt{\alpha}}{\sqrt{1-\alpha}}\sqrt{w'^2_h\hat{\sigma}^2_h + 2w'_h\hat{\sigma}^\top_{hf}w'_f + w'^\top_f\hat{\Sigma}_f w'_f} \le 1 \\
& w'_h + e^\top w'_f > 0,
\end{aligned} \tag{D.7}
$$

Likewise, $\min_{\bar{r}_h\in U_h} (\bar{r}_h - r_f)w'_h = (\hat{r}_h - r_f)w'_h - \delta_h w'_h\hat{\sigma}_h$, and substituting above we get (7). This completes the proof. □

D.2 Continuous ambiguity aversion

Given the MtC formulation (D.2), we use the result of Kamdem (2005) on CVaR formulation of a multivariate t-distribution to write the MtC model as:

$$
\begin{aligned}
\max_{w'\in\mathbb{R}^n_+} \quad & (\bar{r} - r_f e)^\top w' \\
\text{s.t.} \quad & -(\bar{r} - r_f e)^\top w' + es_{\nu,\alpha}\sqrt{w'^\top\hat{\Sigma}w'} \le 1 \\
& e^\top w' > 0.
\end{aligned} \tag{D.8}
$$

Therefore, the continuous ambiguity aversion model is as follows:

$$
\begin{aligned}
\max_{w'\in\mathbb{R}^n_+} \quad & \lambda\Big(\min_{\substack{\bar{r}_h\in U_h\\ \bar{r}_f\in U_f}} (\bar{r} - r_f e)^\top w'\Big) + (1-\lambda)\Big(\max_{\substack{\bar{r}_h\in U_h\\ \bar{r}_f\in U_f}} (\bar{r} - r_f e)^\top w'\Big) \\
\text{s.t.} \quad & \lambda\Big(\max_{\substack{\bar{r}_h\in U_h\\ \bar{r}_f\in U_f}} - (\bar{r} - r_f e)^\top w'\Big) + (1-\lambda)\Big(\min_{\substack{\bar{r}_h\in U_h\\ \bar{r}_f\in U_f}} - (\bar{r} - r_f e)^\top w'\Big) + es_{\nu,\alpha}\sqrt{w'^\top\hat{\Sigma}w'} \le 1 \\
& e^\top w' > 0.
\end{aligned} \tag{D.9}
$$

In terms of home and foreign allocations we re-write (D.9) as:

$$
\begin{aligned}
\max_{(w'_h, w'_f)\in\mathbb{R}^n_+} \quad & \lambda\left(\min_{\bar{r}_h\in U_h}(\bar{r}_h - r_f)w'_h + \min_{\bar{r}_f\in U_f}\ (\bar{r}_f - r_f e)^\top w'_f\right) + \\
& (1-\lambda)\left(\max_{\bar{r}_h\in U_h}(\bar{r}_h - r_f)w'_h + \max_{\bar{r}_f\in U_f}\ (\bar{r}_f - r_f e)^\top w'_f\right) \\
\text{s.t.} \quad & -\lambda\left(\min_{\bar{r}_h\in U_h}(\bar{r}_h - r_f)w'_h + \min_{\bar{r}_f\in U_f}(\bar{r}_f - r_f e)^\top w'_f\right) - \\
& (1-\lambda)\left(\max_{\bar{r}_h\in U_h}(\bar{r}_h - r_f)w'_h + \max_{\bar{r}_f\in U_f}(\bar{r}_f - r_f e)^\top w'_f\right) + \\
& es_{\nu,\alpha}\sqrt{w'^2_h\hat{\sigma}^2_h + 2w'_h\hat{\sigma}^\top_{hf}w'_f + w'^\top_f\hat{\Sigma}_f w'_f} \le 1 \\
& w'_h + e^\top w'_f > 0.
\end{aligned}
$$

One can check that $\min_{\bar{r}_f\in U_f}(\bar{r}_f - r_f e)^\top w'_f$ is equal to $(\hat{r}_f - r_f e)^\top w'_f - \delta_f\sqrt{w'^\top_f\hat{\Sigma}_f w'_f}$ and $\max_{\bar{r}_f\in U_f}(\bar{r}_f - r_f e)^\top w'_f$ is equal to $(\hat{r}_f - r_f e)^\top w'_f + \delta_f\sqrt{w'^\top_f\hat{\Sigma}_f w'_f}$. Substituting above leads to the following formulation:

$$
\begin{aligned}
\max_{(w'_h, w'_f)\in\mathbb{R}^n_+} \quad & \lambda\left(\min_{\bar{r}_h\in U_h}\ (\bar{r}_h - r_f)w'_h + (\hat{r}_f - r_f e)^\top w'_f - \delta_f\sqrt{w'^\top_f\hat{\Sigma}_f w'_f}\right) + \\
& (1-\lambda)\left(\max_{\bar{r}_h\in U_h}(\bar{r}_h - r_f)w'_h + (\hat{r}_f - r_f e)^\top w'_f + \delta_f\sqrt{w'^\top_f\hat{\Sigma}_f w'_f}\right) \\
\text{s.t.} \quad & -\lambda\left(\min_{\bar{r}_h\in U_h}(\bar{r}_h - r_f)w'_h + (\hat{r}_f - r_f e)^\top w'_f - \delta_f\sqrt{w'^\top_f\hat{\Sigma}_f w'_f}\right) - \\
& (1-\lambda)\left(\max_{\bar{r}_h\in U_h}(\bar{r}_h - r_f)w'_h - (\hat{r}_f - r_f e)^\top w'_f + \delta_f\sqrt{w'^\top_f\hat{\Sigma}_f w'_f}\right) + \\
& es_{\nu,\alpha}\sqrt{w'^2_h\hat{\sigma}^2_h + 2w'_h\hat{\sigma}^\top_{hf}w'_f + w'^\top_f\hat{\Sigma}_f w'_f} \le 1 \\
& w'_h + e^\top w'_f > 0,
\end{aligned}
$$

Furthermore, one can easily check that $\min_{\bar{r}_h\in U_h}(\bar{r}_h - r_f)w'_h = (\hat{r}_h - r_f)w'_h - \delta_h w'_h\hat{\sigma}_h$ and

$\max_{\bar{r}_h \in U_h} (\bar{r}_h - r_f)w'_h = (\hat{r}_h - r_f)w'_h + \delta_h w'_h \hat{\sigma}_h$. Replacing these two above, we get:

$$\begin{aligned}
\max_{(w'_h, w'_f) \in \mathbb{R}^n_+} \quad & \lambda \left((\hat{r}_h - r_f)w'_h - \delta_h w'_h \hat{\sigma}_h + (\hat{r}_f - r_f e)^\top w'_f - \delta_f \sqrt{w'^\top_f \hat{\Sigma}_f w'_f} \right) + \\
& (1-\lambda) \left((\hat{r}_h - r_f)w'_h + \delta_h w'_h \hat{\sigma}_h + (\hat{r}_f - r_f e)^\top w'_f + \delta_f \sqrt{w'^\top_f \hat{\Sigma}_f w'_f} \right) \\
\text{s.t.} \quad & -\lambda \left((\hat{r}_h - r_f)w'_h - \delta_h w'_h \hat{\sigma}_h + (\hat{r}_f - r_f e)^\top w'_f - \delta_f \sqrt{w'^\top_f \hat{\Sigma}_f w'_f} \right) - \\
& (1-\lambda) \left((\hat{r}_h - r_f)w'_h + \delta_h w'_h \hat{\sigma}_h + (\hat{r}_f - r_f e)^\top w'_f + \delta_f \sqrt{w'^\top_f \hat{\Sigma}_f w'_f} \right) + \\
& es_{\nu,\alpha} \sqrt{w'^2_h \hat{\sigma}^2_h + 2w'_h \hat{\sigma}^\top_{hf} w'_f + w'^\top_f \hat{\Sigma}_f w'_f} \leq 1 \\
& w'_h + e^\top w'_f > 0.
\end{aligned}$$

Gathering similar terms together, we obtain (13). This completes the proof. □

D.3 Worst-case ambiguity aversion with interval ambiguity

We first define the interval ambiguity set of the mean returns and then present the second-order cone program of worst-case ambiguity aversion model.

Definition D.1 (Interval ambiguity)**.** *Mean returns belong to the interval set:*

$$U_I = \{\bar{r} \in \mathbb{R}^n \mid \bar{r}_- \leq \bar{r} \leq \bar{r}_+\},$$

where $\bar{r}_-$ and $\bar{r}_+$ are given vectors, and the inequalities are component-wise.

The worst-case MtC maximization is as follows:

$$\max_{w \in \mathbb{X}} \min_{\bar{r} \in U_I} \min_{\pi \in \mathbb{D}} \frac{\mathbb{E}(\tilde{r}_p - r_f)}{\text{CVaR}(\tilde{r}_p - r_f)}. \tag{D.10}$$

The following theorem provides the second-order cone program formulation.

Theorem D.3 (Second-order cone program worst-case model with interval ambiguity)**.** *Model (D.10) is cast as:*

$$\begin{aligned}
\max_{v'_-, v'_+ \in \mathbb{R}^n_+} \quad & (\bar{r}_- - r_f e)^\top v'_- - (\bar{r}_+ - r_f e)^\top v'_+ \\
\text{s.t.} \quad & (\bar{r}_+ - r_f e)^\top v'_+ - (\bar{r}_- - r_f e)^\top v'_- + \frac{\alpha}{1-\alpha} \sqrt{(v'_- - v'_+)^\top \hat{\Sigma} (v'_- - v'_+)} \leq 1 \\
& v'_- - v'_+ \geq 0 \\
& e^\top (v'_- - v'_+) > 0.
\end{aligned} \tag{D.11}$$

From the optimal solutions v'^*_+ and v'^*_-, we obtain the solution to (D.10) as $w^* = \frac{1}{e^\top v'*} v'*$

where $v'* = v'^*_- - v'^*_+$.

Proof

Given the MtC formulation (D.2), the worst-case MtC model is as follows:

$$\begin{aligned}
\max_{w'\in\mathbb{R}^n_+} \quad & \min_{\bar{r}\in U_I}\min_{\pi\in\mathbb{D}} \ (\mathbb{E}(\tilde{r}) - r_f e)^\top w' \\
\text{s.t.} \quad & \max_{\bar{r}\in U_I}\max_{\pi\in\mathbb{D}} \ \mathrm{CVaR}((\tilde{r} - r_f e)^\top w') \le 1 \\
& e^\top w' > 0.
\end{aligned} \tag{D.12}$$

Replacing CVaR from the fundamental minimization formula (Appendix C.1) we have:

$$\begin{aligned}
\max_{w'\in\mathbb{R}^n_+} \quad & \min_{\bar{r}\in U_I}\min_{\pi\in\mathbb{D}} \ (\mathbb{E}(\tilde{r}) - r_f e)^\top w' \\
\text{s.t.} \quad & \max_{\bar{r}\in U_I}\max_{\pi\in\mathbb{D}}\min_{\gamma\in\mathbb{R}} \ F((\tilde{r} - r_f e)^\top w', \gamma) \le 1 \\
& e^\top w' > 0.
\end{aligned} \tag{D.13}$$

Obviously, $\min_{\pi\in\mathbb{D}} (\mathbb{E}(\tilde{r}) - r_f e)^\top w' = (\bar{r} - r_f e)^\top w'$. Further, the max-min optimization in the first constraint can be obtained from Proposition 1 in Lotfi and Zenios (2018). Therefore, the above formulation can be written as follows:

$$\begin{aligned}
\max_{w'\in\mathbb{R}^n_+} \quad & \min_{\bar{r}\in U_I} (\bar{r} - r_f e)^\top w' \\
\text{s.t.} \quad & -\left(\min_{\bar{r}\in U_I} (\bar{r} - r_f e)^\top w'\right) + \frac{\sqrt{\alpha}}{\sqrt{1-\alpha}}\sqrt{w'^\top \Sigma w'} \le 1 \\
& e^\top w' > 0.
\end{aligned} \tag{D.14}$$

One can check $\min_{\bar{r}\in U_I} (\bar{r} - r_f e)^\top w' = \max_{v'_-, v'_+\in\mathbb{R}^n_+} (\bar{r}_- - r_f e)^\top v'_- - (\bar{r}_+ - r_f e)^\top v'_+$ with $w' = v'_- - v'_+$. This leads us to the following formulation.

$$\begin{aligned}
\max_{v'_-, v'_+\in\mathbb{R}^n_+} \quad & (\bar{r}_- - r_f e)^\top v'_- - (\bar{r}_+ - r_f e)^\top v'_+ \\
\text{s.t.} \quad & (\bar{r}_+ - r_f e)^\top v'_+ - (\bar{r}_- - r_f e)^\top v'_- + \frac{\alpha}{1-\alpha}\sqrt{(v'_- - v'_+)^\top \hat{\Sigma} (v'_- - v'_+)} \le 1 \\
& v'_- - v'_+ \ge 0 \\
& e^\top (v'_- - v'_+) > 0.
\end{aligned} \tag{D.15}$$

This completes the proof. □

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